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# Regularization Schemes for Null-Surface Quantization of Field Theories[⋆]

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## ABSTRACT

In this paper we analyze different regularization schemes used in calculating the fermion and photon self-energy. We find that the infrared regularization introduced previously protects the photon self-energy from the ultraviolet quadratic divergence and that the presence of **two** Hamiltonians is crucial for this cancellation. We also find that global cut-off regularization does not cancel the quadratically ultraviolet divergences, making it undesirable as a regularization scheme.

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## 1. Introduction

In recent papers[1] [2] [3] [4] we studied $QED$ in null-surface quantization, which treats $x^+$ and $x^-$ on equal footing[5] . Our previous work [1], [2] obtained the same results for fermion and photon one-loop calculations using transverse dimensional regularization as the canonical quantization. In the present paper we study different regularization schemes in calculating the fermion self-energy and photon self-energy to one-loop in $QED$ in our present null-surface quantization method. We study transverse longitudinal cut-off, Pauli-Villars regularization and global cut-off regularization. We concentrate on ultraviolet regularization, since Harindranath and Zhang[6] have studied the question of infrared regularization extensively and carefully their first Physical Review paper in [6] and we don't add anything else to that work. The outline of the paper is as follows: we first review the perturbative solutions of $QED$; we then redo the fermion self-energy and photon self-energy and wave-function renormalization in transverse momentum regularization; we then calculate the same quantities on Pauli-Villars regularization; finally, we conclude with using global cut-off regularization for the fermion and photon self-energies and wave-function renormalization.

## 2. Perturbative Solution to QED

Let us recall that the Hamiltonian $P^-$ was constructed out of the Hamiltonian density $\mathcal{H}$. But first, we must express all the fields in terms of the fields defined on $x^+ = 0$. We have

$$\mathcal{H} = \{\frac{1}{2}(B_P^{-2} + E_P^{-2}) + \psi_+^\dagger \psi_+ e A_P^- + \psi_+^\dagger \gamma_0 \gamma_i \frac{i\partial_i}{2}\psi_- - e\psi_+^\dagger \gamma_0 \gamma_i \frac{A_P^i}{2}\psi_-$$

$$+\psi_-^\dagger \gamma_0 \gamma_i \frac{i\partial_i}{2}\psi_+ - e\psi_-^\dagger \gamma_0 \gamma_i \frac{A_P^i}{2}\psi_+ + m\psi_+^\dagger \frac{\gamma^-}{4}\psi_- + m\psi_-^\dagger \frac{\gamma^+}{4}\psi_+\} \tag{2.1}$$

where

$$B_P^- = (\partial^1 A_P^2 - \partial^2 A_P^1) \tag{2.2}$$

and

$$E_P^- = -\frac{1}{2}\partial^+ A_P^- \tag{2.3}$$

We follow the two-component notation of Harindranath and Zhang [6] and observe that the fermionic fields are

$$\psi_+^\dagger = (\xi^\dagger, 0) \tag{2.4}$$

and

$$\psi_-^\dagger = (0, \eta^\dagger) \tag{2.5}$$

so that we have

$$\eta = \frac{1}{i\partial^+}\{i\partial_i\sigma_i - \sigma_i e A_P^i + im\}\xi \tag{2.6}$$

where $\sigma_i$ are the 2x2 Pauli sigma matrices and now both spinors $\xi$ and $\eta$ are two-component spinors [6].

Substituting this in the equation [(2.1)] we get

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I \tag{2.7}$$

where

$$\mathcal{H}_0 = \frac{1}{2}A_P^i(\partial_i^2)A_P^i + \frac{1}{2}\xi^\dagger\frac{(-\partial_i^2 + m^2)}{i\partial^+}\xi \tag{2.8}$$

while the interaction Hamiltonian density is :

$$\mathcal{H}_I = \mathcal{H}_{e^+e^-A} + \mathcal{H}_{e^+e^-AA} + \mathcal{H}_{e^+e^-e^+e^-} \tag{2.9}$$

These pieces are as follows:

$$\mathcal{H}_{e^+e^-A} = -e\xi^\dagger\frac{i\partial_k}{i\partial^+}A_P^k\xi + \frac{e}{2}\xi^\dagger\sigma_i A_P^i\frac{1}{i\partial^+}(\sigma_j i\partial_j + im)\xi + \frac{e}{2}\xi^\dagger\frac{1}{i\partial^+}(\sigma_j i\partial_j - im)\sigma_i A_P^i\xi$$

$$-e\xi^\dagger i\partial_k A_P^k\frac{1}{i\partial^+}\xi + \frac{e}{2}\xi^\dagger\sigma_i(\sigma_j i\partial_j + im)\frac{1}{i\partial^+}A_P^i\xi + \frac{e}{2}\xi^\dagger\sigma_i A_P^i(\sigma_j i\partial_j - im)\frac{1}{i\partial^+}\xi$$

$$-2e\xi^\dagger\xi\frac{1}{i\partial^+}i\partial_i A_P^i \tag{2.10}$$

$$\mathcal{H}_{e^+e^-AA} = e^2\xi^\dagger\sigma_i A_P^i\frac{1}{i\partial^+}\sigma_j A_P^j\xi \tag{2.11}$$

$$\mathcal{H}_{e^+e^-e^+e^-} = 4e^2\xi^\dagger\xi\frac{1}{(i\partial^+)^2}\xi^\dagger\xi - 2e^2\frac{1}{(i\partial^+)}\xi^\dagger\xi\frac{1}{(i\partial^+)}\xi^\dagger\xi \tag{2.12}$$

We see that we have recovered Harindranath and Zhang's results, but with a difference [6]: where they had just the term corresponding to their $\Gamma^i_{q0}(p_1, p_2, k)$, we have $\frac{1}{2}$ of this **plus** $\frac{1}{2}$ of its complex conjugate. Note also that our result is real, compared to Harindranath and Zhang's result, which is complex, since they lack the complex conjugate piece.

## 3. Transverse Momentum Regularization

We use now transverse momentum regularization to regulate the ultraviolet divergence which we might encounter in calculating the electron and photon self-energies.

First, we look at the fermion self-energy. Since both $P^+$ and $P^-$ are dynamical, we get contributions from both of them. First, let us compute the contributions from $P^-$, denoted by $\delta P^-$, for an electron with momentum $p$ :

$$\delta P^- = \delta P_1^- + \delta P_2^- + \delta P_3^- \tag{3.1}$$

where

$$\delta P_1^- = (-e)^2\int\frac{d^2k_\perp}{16\pi^3}dk^+\frac{\theta(k^+)\theta(p^+-k^+)}{[k^+]}\frac{1}{4}(\Gamma^i_{q0}(p, p-k, k) + \Gamma^i_{q0}(p-k, p, k))^2$$

$$\frac{1}{p^- - k^- - (p-k)^-} \tag{3.2}$$

which is due to the emission and reabsorption of a virtual photon ,coming from the iteration of $\mathcal{H}_{e^+e^-A}$. The notation $\frac{1}{[x]}$ is the method of regularizing the infrared behaviour as follows: $\frac{1}{[x]} = \frac{1}{2}\{\frac{1}{x+i\epsilon} + \frac{1}{x-i\epsilon}\}$, as Harindranath and Zhang used [6].

The second piece comes from the four lepton piece of the Hamiltonian, $\mathcal{H}_{e^+e^-e^+e^-}$, where two of the fermions have been 'tied' together

$$\delta P_2^- = 2e^2 \int \frac{d^2k_\perp}{16\pi^3} dk^+\theta(k^+)\{\frac{1}{[p^+-k^+]^2} - \frac{1}{[p^++k^+]^2}\} \tag{3.3}$$

The last piece comes from the two-photon piece of the Hamiltonian, $\mathcal{H}_{e^+e^-AA}$, where the photon lines have been 'tied' together

$$\delta P_3^- = e^2 \int \frac{d^2k_\perp}{16\pi^3} \frac{dk^+\theta(k^+)}{[k^+]} Tr \frac{\sigma_i\sigma^i}{[p^+-k^+]} \tag{3.4}$$

The first contribution becomes, after introducing the parameters $[x] = \frac{[k^+]}{p^+}$ and $\kappa_\perp = k_\perp - xp_\perp$ :

$$\delta P_1^- = \frac{e^2}{p^+} \int \frac{d(\kappa^2)}{16\pi^2} \int\limits_0^1 dx \frac{1}{[1-x]} \frac{\kappa^2(\frac{4}{[x]^2} - \frac{4}{[x]} + 2)}{x(1-x)p^2 - \kappa^2 - xm^2} \tag{3.5}$$

We perform now transverse cut-off regularization, $|\kappa_\perp^2| < \Lambda_\perp^2$ and obtain:

$$\delta P_1^- = -\frac{e^2}{16\pi^2}\{\frac{p^2-m^2}{p^+}[ln\frac{\Lambda_\perp^2}{m^2}(2ln\frac{p^+}{\lambda} - \frac{3}{2}) - \int\limits_0^1 dx(\frac{2}{[x]} - 2 + x)ln(x - x(1-x)\frac{p^2}{m^2})]$$

$$-2\frac{p^2}{p^+}[\frac{3}{2}ln\frac{\Lambda_\perp^2}{m^2} - \int\limits_0^1 dx(2-x)ln(x-x(1-x)\frac{p^2}{m^2})] + 2\frac{\Lambda_\perp^2}{p^+}(\frac{\pi p^+}{2\lambda} - 1 + ln\frac{p^+}{\lambda})\} \tag{3.6}$$

where $\lambda$ is the infrared regularization.

The second contribution, $\delta P_2^-$ becomes

$$\delta P_2^- = \frac{2e^2}{p^+}\int \frac{d^2k_\perp}{16\pi^2}\int_0^\infty dx(\frac{1}{[1-x]^2} - \frac{1}{(1+x)^2}) \tag{3.7}$$

Using transverse cut-off regularization, this is :

$$\delta P_2^- = \frac{e^2}{8\pi^2}\frac{\Lambda_\perp^2}{p^+}(\frac{\pi p^+}{2\lambda} - 1) \tag{3.8}$$

The last contribution, $\delta P_3^-$ becomes

$$\delta P_3^- = \frac{e^2}{p^+}\int \frac{d^2k_\perp}{16\pi^2}\int_0^\infty \frac{dx}{[x]}\frac{2}{[1-x]} \tag{3.9}$$

Using transverse cut-off regularization, this is:

$$\delta P_3^- = \frac{e^2}{8\pi^2}\frac{\Lambda_\perp^2}{p^+}ln\frac{p^+}{\lambda} \tag{3.10}$$

The total contribution to $\delta P^-$ is

$$\delta P^- = -\frac{e^2}{16\pi^2}\{\frac{p^2-m^2}{p^+}[ln\frac{\Lambda_\perp^2}{m^2}(2ln\frac{p^+}{\lambda} - \frac{3}{2}) - \int_0^1 dx(\frac{2}{[x]} - 2 + x)ln(x - x(1-x)\frac{p^2}{m^2})]$$

$$-2\frac{p^2}{p^+}[\frac{3}{2}ln\frac{\Lambda_\perp^2}{m^2} - \int_0^1 dx(2-x)ln(x - x(1-x)\frac{p^2}{m^2})]\} \tag{3.11}$$

Note the fact that the overall $\delta P^-$ is free of ultraviolet quadratic divergences, like in the work of Harindranath and Zhang [6]. The piece of the quadratic ultraviolet divergence proportional to the linear infrared piece is exactly canceled by the photon instantaneous piece, while the logarithmic infrared divergence is exactly canceled by the fermion instantaneous piece. As Harindranath and Zhang pointed out in their paper [6], this is possible due to the $k^+$ regularization which we use.

We compute now easily the contributions from $\delta P^+$, by using the explicit parity symmetry of our formalism:

$$\delta P^+ = -\frac{e^2}{16\pi^2}\{\frac{p^2-m^2}{p^-}[ln\frac{\Lambda_\perp^2}{m^2}(2ln\frac{p^-}{\lambda}-\frac{3}{2})-\int_0^1 dx(\frac{2}{[x]}-2+x)ln(x-x(1-x)\frac{p^2}{m^2})]$$

$$-2\frac{p^2}{p^-}[\frac{3}{2}ln\frac{\Lambda_\perp^2}{m^2}-\int_0^1 dx(2-x)ln(x-x(1-x)\frac{p^2}{m^2})]\} \qquad (3.12)$$

Putting all of this together, we get

$$\delta M^2 = p^+(\delta P^-)_{p^2=m^2=p^+p^-} + p^-(\delta P^+)_{p^2=m^2=p^+p^-}$$

$$= \frac{3e^2}{8\pi^2}m^2 ln\frac{\Lambda_\perp^2}{m^2} \qquad (3.13)$$

which is identical to the usual canonical quantization result [7] . We also get the same coefficient as in the canonical calculation. Notice that we get the same result as in dimensional regularization [2], [3].

We can also obtain now the fermion wave-function renormalization as calculated in the previous work [2]. It is

$$Z_2^{-1} = 1-(\frac{\partial\delta P^-}{\partial p^-})_{p^2=m^2=p^+p^-} - (\frac{\partial\delta P^+}{\partial p^+})_{p^2=m^2=p^+p^-}$$

$$Z_2^{-1} = 1-\frac{e^2}{4\pi^2}\{ln\frac{\Lambda_\perp^2}{m^2}(3-ln\frac{m^2}{\lambda^2})\} \qquad (3.14)$$

We look now at the one-loop quantum correction to the photon mass and wave-function . We proceed like we did above, and do the transverse cut-off regularization.

We get one contribution from the piece of the Hamiltonian $\mathcal{H}_{e^+e^-A}$ plus a contribution from the piece $\mathcal{H}_{e^+e^-AA}$:

$$\delta P^- = \delta P_1^- + \delta P_2^- \tag{3.15}$$

For a photon of energy $q$ we get the following from the iteration of $\mathcal{H}_{e^+e^-A}$ (the overall minus sign comes from the fermion loop):

$$\delta P_1^- = (-1)\frac{(-2e)^2}{q^+}\int \frac{d^2k_\perp}{16\pi^3}dk^+\theta(k^+)\theta(q^+-k^+)Tr\{\chi^\dagger\chi\frac{q_i}{q^+}\frac{q^i}{q^+}\chi^\dagger\chi\frac{1}{q^- - k^- - (q-k)^-}\} \tag{3.16}$$

which becomes, upon using $[x] = \frac{[k^+]}{q^+}$ and $\kappa_\perp = k_\perp - xq_\perp$

$$\delta P_1^- = -\frac{4e^2}{q^+}\int \frac{d(\kappa^2)}{16\pi^2}\int_0^1 dx \frac{(1-x)xq_\perp^2}{x(1-x)q^2 - \kappa^2 - m^2} \tag{3.17}$$

Using the cut-off $|\kappa_\perp^2| < \Lambda_\perp^2$, we get

$$\delta P_1^- = \frac{e^2}{16\pi^2}\frac{q^+q^- - q^2}{q^+}\frac{4}{6}ln\frac{\Lambda_\perp^2}{m^2} \tag{3.18}$$

where the meaning of $\lambda$ is as before.

For the second contribution, we have as follows after tying together $\mathcal{H}_{e^+e^-AA}$:

$$\delta P_2^- = e^2\int \frac{d^2k_\perp}{16\pi^3}\frac{dk^+\theta(k^+)}{[k^+]}\frac{Tr\sigma_i\sigma^i}{[q^+ - k^+]} \tag{3.19}$$

which becomes, upon using $[x] = \frac{[k^+]}{q^+}$ and $\kappa_\perp = k_\perp - xq_\perp$

$$\delta P_2^- = \frac{e^2}{q^+}\int \frac{d^2k_\perp}{16\pi^2}\int_0^\infty \frac{dx}{[x]}\frac{2}{[1-x]} \tag{3.20}$$

Using transverse cut-off regularization, we get:

$$\delta P_2^- = \frac{e^2}{q^+}\frac{\Lambda_\perp^2}{8\pi^2} ln\frac{q^+}{\lambda} \tag{3.21}$$

The total contribution to $\delta P^-$ is

$$\delta P^- = \frac{e^2}{24\pi^2}\frac{q^+q^- - q^2}{q^+} ln\frac{\Lambda_\perp^2}{m^2} + \frac{e^2}{q^+}\frac{\Lambda_\perp^2}{8\pi^2} ln\frac{q^+}{\lambda} \tag{3.22}$$

We get a similar result for $\delta P^+$ :

$$\delta P^+ = \frac{e^2}{24\pi^2}\frac{q^-q^+ - q^2}{q^-} ln\frac{\Lambda_\perp^2}{m^2} + \frac{e^2}{q^-}\frac{\Lambda_\perp^2}{8\pi^2} ln\frac{q^-}{\lambda} \tag{3.23}$$

The correction to the photon mass becomes then

$$\delta\mu^2 = q^+(\delta P^-)_{q^2=0=q^+q^-} + q^-(\delta P^+)_{q^2=0=q^+q^-}$$

$$\delta\mu^2 = e^2\frac{\Lambda_\perp^2}{8\pi^2} ln\frac{q^+q^-}{\lambda^2} \tag{3.24}$$

The one-loop correction to the photon mass vanishes if $q^+q^-$ goes to zero and the infrared cut-off $\lambda$ also going to zero at the same rate. It is precisely this infrared cancelation which protects the photon mass from being quadratically ultraviolet divergent to one-loop, in this regularization. Note that it is crucial that we get contributions from both $P^-$ **and** $P^+$. In other light-cone quantizations, the absence of the second contribution will give rise to a mixed quadratic ultraviolet and logarithmic infrared singularity (for instance, [6]).

We get now the photon wave-function renormalization by simply using the formula introduced before [2] :

$$Z_3^{-1} = 1 - (\frac{\partial \delta P^-}{\partial q^-})_{q^2=0} - (\frac{\partial \delta P^+}{\partial q^+})_{q^2=0}$$

$$Z_3^{-1} = 1 + \frac{e^2}{12\pi^2} ln \frac{\Lambda_\perp^2}{m^2} \tag{3.25}$$

This also agrees with the usual canonical results [7] , and insures that we get the same beta-function for the photon as well, as in previous calculations [2], [3].

## 4. Pauli-Villars Regularization

We use now Pauli-Villars regularization to regulate the ultraviolet divergence which we might encounter in calculating the electron and photon self-energies. This means that we subtract from the ultraviolet regulated integrands with total internal field on mass shell the same integrand with total momentum $\kappa^2 = M^2$, and we take the large field mass $M$ to $\infty$ at the end of the calculation.

First, we look at the fermion self-energy. The first contribution becomes, after introducing the parameters $[x] = \frac{[k^+]}{p^+}$ and $\kappa_\perp = k_\perp - xp_\perp$ :

$$\delta P_1^- = \frac{e^2}{p^+} \int \frac{d(\kappa^2)}{16\pi^2} \int_0^1 dx \{ \frac{1}{[1-x]} \frac{\kappa^2(\frac{4}{[x]^2} - \frac{4}{[x]} + 2)}{x(1-x)p^2 - \kappa^2 - xm^2}$$

$$- \frac{1}{[1-x]} \frac{\kappa^2(\frac{4}{[x]^2} - \frac{4}{[x]} + 2)}{x(1-x)p^2 - \kappa^2 - (1-x)M^2 - xm^2} \} \tag{4.1}$$

Upon integrating, we obtain:

$$\delta P_1^- = -\frac{e^2}{16\pi^2} \{ \frac{p^2 - m^2}{p^+} [ln \frac{M^2}{m^2} (2 ln \frac{p^+}{\lambda} - \frac{3}{2}) - \int_0^1 dx (\frac{2}{[x]} - 2 + x) ln(x - x(1-x)\frac{p^2}{m^2})$$

$$+\int_0^1 dx(\frac{2}{[x]}-2+x)ln(1-x-x(1-x)\frac{p^2}{M^2})]$$

$$-2\frac{p^2}{p^+}[\frac{3}{2}ln\frac{M^2}{m^2}-\int_0^1 dx(2-x)ln(x-x(1-x)\frac{p^2}{m^2})+\int_0^1 dx(2-x)ln(1-x-x(1-x)\frac{p^2}{M^2})] \qquad (4.2)$$

where $\lambda$ is the infrared regularization.

The second contribution, $\delta P_2^-$ becomes

$$\delta P_2^- = \frac{2e^2}{p^+}\int\frac{d^2k_\perp}{8\pi^2}\int_0^\infty dx\{(\frac{1}{[1-x]^2}-\frac{1}{(1+x)^2})-(\frac{1}{[1-x]^2}-\frac{1}{(1+x)^2})\} \qquad (4.3)$$

This is identically zero.

The last contribution, $\delta P_3^-$ becomes

$$\delta P_3^- = \frac{e^2}{p^+}\int\frac{d^2k_\perp}{8\pi^2}\int_0^\infty dx\{\frac{2}{[x][1-x]}-\frac{2}{[x][1-x]}\} \qquad (4.4)$$

This is also identically zero.

The total contribution to $\delta P^-$ is

$$\delta P^- = -\frac{e^2}{16\pi^2}\{\frac{p^2-m^2}{p^+}[ln\frac{M^2}{m^2}(2ln\frac{p^+}{\lambda}-\frac{3}{2})-\int_0^1 dx(\frac{2}{[x]}-2+x)ln(x-x(1-x)\frac{p^2}{m^2})$$

$$+\int_0^1 dx(\frac{2}{[x]}-2+x)ln(1-x-x(1-x)\frac{p^2}{M^2})]$$

$$-2\frac{p^2}{p^+}[\frac{3}{2}ln\frac{M^2}{m^2}-\int_0^1 dx(2-x)ln(x-x(1-x)\frac{p^2}{m^2})+\int_0^1 dx(2-x)ln(1-x-x(1-x)\frac{p^2}{M^2})\} \qquad (4.5)$$

We get the same result as in transverse dimensional regularization [2], [3] and transverse cut-off regularization: we have a quantity which is log ultraviolet divergent but infrared finite, with $M$ playing the role of ultraviolet cut-off.

We compute now easily the contributions from $\delta P^+$, by using the explicit parity symmetry of our formalism:

$$\delta P^+ = -\frac{e^2}{16\pi^2}\{\frac{p^2-m^2}{p^-}[ln\frac{M^2}{m^2}(2ln\frac{p^-}{\lambda}-\frac{3}{2}) - \int_0^1 dx(\frac{2}{[x]}-2+x)ln(x-x(1-x)\frac{p^2}{m^2})$$

$$+\int_0^1 dx(\frac{2}{[x]}-2+x)ln(1-x-x(1-x)\frac{p^2}{M^2})]$$

$$-2\frac{p^2}{p^-}[\frac{3}{2}ln\frac{M^2}{m^2} - \int_0^1 dx(2-x)ln(x-x(1-x)\frac{p^2}{m^2}) + \int_0^1 dx(2-x)ln(1-x-x(1-x)\frac{p^2}{M^2})]\} \tag{4.6}$$

Putting all of this together, we get

$$\delta M^2 = p^+(\delta P^-)_{p^2=m^2=p^+p^-} + p^-(\delta P^+)_{p^2=m^2=p^+p^-}$$

$$= \frac{3e^2}{8\pi^2}m^2 ln\frac{M^2}{m^2} \tag{4.7}$$

just like above, with $M$ playing the role of the ultraviolet cut-off.

We can also obtain now the fermion wave-function renormalization as calculated in the previous work [2]. We get

$$Z_2^{-1} = 1 - (\frac{\partial \delta P^-}{\partial p^-})_{p^2=m^2=p^+p^-} - (\frac{\partial \delta P^+}{\partial p^+})_{p^2=m^2=p^+p^-}$$

$$Z_2^{-1} = 1 - \frac{e^2}{4\pi^2}\{ln\frac{M^2}{m^2}(3 - ln\frac{m^2}{\lambda^2})\} \tag{4.8}$$

We look now at the one-loop quantum correction to the photon mass and wave-function . We proceed like we did above, and do the Pauli-Villars regularization.

We get one contribution from the piece of the Hamiltonian $\mathcal{H}_{e^+e^-A}$ plus a contribution from the piece $\mathcal{H}_{e^+e^-AA}$:

$$\delta P^- = \delta P_1^- + \delta P_2^- \tag{4.9}$$

For a photon of energy $q$ we get the following from the iteration of $\mathcal{H}_{e^+e^-A}$ (the overall minus sign comes from the fermion loop):

$$\delta P_1^- = (-1)\frac{(-2e)^2}{q^+}\int \frac{d^2k_\perp}{16\pi^3}dk^+\theta(k^+)\theta(q^+ - k^+)Tr\{\chi^\dagger\chi\frac{q_i}{q^+}\frac{q^i}{q^+}\chi^\dagger\chi\frac{1}{q^- - k^- - (q-k)^-}\} \tag{4.10}$$

which becomes, upon using $[x] = \frac{[k^+]}{q^+}$ and $\kappa_\perp = k_\perp - xq_\perp$ and performing a Pauli-Villars subtraction:

$$\delta P_1^- = -\frac{4e^2}{q^+}\int \frac{d(\kappa^2)}{16\pi^2}\int_0^1 dx\{\frac{(1-x)xq_\perp^2}{x(1-x)q^2 - \kappa^2 - m^2} - \frac{(1-x)xq_\perp^2}{x(1-x)q^2 - \kappa^2 - M^2}\} \tag{4.11}$$

Upon integration, we get:

$$\delta P_1^- = \frac{e^2}{16\pi^2}\frac{q^+q^- - q^2}{q^+}\frac{4}{6}ln\frac{M^2}{m^2} \tag{4.12}$$

where the meaning of $M$ is as before.

For the second contribution, we have as follows after tying together $\mathcal{H}_{e^+e^-AA}$:

$$\delta P_2^- = e^2\int \frac{d^2k_\perp}{16\pi^3}\frac{dk^+\theta(k^+)}{[k^+]}\frac{Tr\sigma_i\sigma^i}{[q^+ - k^+]} \tag{4.13}$$

which becomes, upon using $[x] = \frac{[k^+]}{q^+}$ and $\kappa_\perp = k_\perp - xq_\perp$

$$\delta P_2^- = \frac{e^2}{q^+}\int \frac{d^2k_\perp}{8\pi^2}\int_0^\infty \frac{dx}{[x]}\frac{2}{[1-x]} \tag{4.14}$$

Using a Pauli-Villars subtraction , we get:

$$\delta P_2^- = \frac{e^2}{q^+}\frac{\Lambda_\perp^2}{8\pi^2}\int_0^\infty dx\{\frac{2}{[x][1-x]} - \frac{2}{[x][1-x]}\} \tag{4.15}$$

This is just

$$\delta P_2^- = 0 \tag{4.16}$$

The total contribution to $\delta P^-$ is

$$\delta P^- = \frac{e^2}{24\pi^2}\frac{q^+q^- - q^2}{q^+} ln\frac{M^2}{m^2} \tag{4.17}$$

We get a similar result for $\delta P^+$ :

$$\delta P^+ = \frac{e^2}{24\pi^2}\frac{q^-q^+ - q^2}{q^-} ln\frac{M^2}{m^2} \tag{4.18}$$

The correction to the photon mass becomes then

$$\delta\mu^2 = q^+(\delta P^-)_{q^2=0=q^+q^-} + q^-(\delta P^+)_{q^2=0=q^+q^-}$$

$$\delta\mu^2 = 0 \tag{4.19}$$

The one-loop correction to the photon mass vanishes identically, just like in the previous results.

We get now the photon wave-function renormalization by simply using the formula introduced before [2] :

$$Z_3^{-1} = 1 - (\frac{\partial\delta P^-}{\partial q^-})_{q^2=0} - (\frac{\partial\delta P^+}{\partial q^+})_{q^2=0}$$

$$Z_3^{-1} = 1 + \frac{e^2}{12\pi^2} ln\frac{M^2}{m^2} \tag{4.20}$$

This also agrees with the usual canonical results [7] , with the large Pauli-Villars mass playing the role of ultraviolet cut-off.

## 5. Global Cut-Off Regularization

We use now global cut-off regularization to regulate the ultraviolet divergence which we might encounter in calculating the electron and photon self-energies. This means that we cut-off $\kappa_\perp$ such that the total $p^-$ is less than some global cut-off $\Lambda$ [6]

$$\frac{\kappa_\perp^2}{[x]} + \frac{\kappa_\perp^2 + m^2}{[1-x]} \leq \Lambda^2, \lambda^2 >> \kappa_\perp^2, m^2 \tag{5.1}$$

This means that there is a maximum $\kappa_{max}$

$$k_{max}^2 = x(1-x)\Lambda^2 - xm^2 \tag{5.2}$$

so that all $\kappa_\perp$ integrals are bounded above by this $\kappa_{max}$. This also means that $x$ is regularized differently

$$0 \leq x \leq x_{max} = 1 - \frac{m^2}{\Lambda^2} \tag{5.3}$$

First, we look at the fermion self-energy. We have as before :

$$\delta P_1^- = -\frac{e^2}{16\pi^2}\{\frac{p^2-m^2}{p^+}[\int_0^{x_{max}} dx \int_0^{\kappa_{max}} d\kappa_\perp^2 \frac{\frac{2}{[x]} - 2 + x}{\kappa_\perp^2 + (xm^2 - x(1-x)p^2)}]$$

$$-2\frac{p^2}{p^+}[\int_0^{x_{max}} dx \int_0^{\kappa_{max}} d\kappa_\perp^2 \frac{2-x}{\kappa_\perp^2 + (xm^2 - x(1-x)p^2)}] + \int_0^{x_{max}} dx \int_0^{\kappa_{max}} d\kappa_\perp^2 (\frac{4}{[x]} - 4 + 2x) \tag{5.4}$$

This becomes

$$\delta P_1^- = -\frac{e^2}{16\pi^2}\{\frac{p^2-m^2}{p^+}[ln\frac{\Lambda^2}{m^2}(2ln\frac{m^2}{\Lambda^2} - \frac{3}{2}) - \int_0^{x_{max}} dx(\frac{2}{[x]} - 2 + x)ln(x - x(1-x)\frac{p^2}{m^2})]$$

$$-2\frac{p^2}{p^+}[\frac{3}{2}ln\frac{\Lambda^2}{m^2}-\int\limits_0^{x_{max}} dx(2-x)ln(x-x(1-x)\frac{p^2}{m^2})]+4\int\limits_0^{x_{max}}\frac{dx}{[x]^2}\int\limits_0^{\kappa_{max}} d\kappa_\perp^2+2\int\limits_0^{x_{max}}\frac{dx}{[x]}\int\limits_0^{\kappa_{max}} d\kappa_\perp^2\} \tag{5.5}$$

The second contribution, $\delta P_2^-$ becomes

$$\delta P_2^- = \frac{2e^2}{p^+}\int\limits_0^{\kappa_{max}}\frac{d\kappa_\perp^2}{16\pi^2}\int\limits_0^{\infty} dx(\frac{1}{[1-x]^2}-\frac{1}{(1+x)^2}) \tag{5.6}$$

This can be re-written as

$$\delta P_2^- = \frac{4e^2}{p^+}\int\limits_0^{\kappa_{max}}\frac{d\kappa_\perp^2}{16\pi^2}\int\limits_0^{x_{max}}\frac{dx}{[x]^2} \tag{5.7}$$

to make the cancellation with terms from $\delta P_1^-$ more obvious.

The last contribution, $\delta P_3^-$ becomes

$$\delta P_3^- = \frac{e^2}{p^+}\int\limits_0^{\kappa_{max}}\frac{d\kappa_\perp^2}{16\pi^2}\int\limits_0^{\infty} dx\frac{2}{[x][1-x]} \tag{5.8}$$

This can also be re-written as

$$\delta P_3^- = \frac{2e^2}{p^+}\int\limits_0^{\kappa_{max}}\frac{d\kappa_\perp^2}{16\pi^2}\int\limits_0^{x_{max}}\frac{dx}{[x]} \tag{5.9}$$

The total contribution to $\delta P^-$ is

$$\delta P^- = -\frac{e^2}{16\pi^2}\{\frac{p^2-m^2}{p^+}[ln\frac{\Lambda^2}{m^2}(2ln\frac{m^2}{\Lambda^2}-\frac{3}{2})-\int\limits_0^{x_{max}} dx(\frac{2}{[x]}-2+x)ln(x-x(1-x)\frac{p^2}{m^2})]$$

$$-2\frac{p^2}{p^+}[\frac{3}{2}ln\frac{\Lambda^2}{m^2}-\int\limits_0^{x_{max}} dx(2-x)ln(x-x(1-x)\frac{p^2}{m^2})]\} \tag{5.10}$$

Notice that the last two terms of $\delta P_1^-$ were cancelled by $\delta P_2^-$ and $\delta P_3^-$ respectively, similar to the situation in transverse cut-off regularization. We get the same result as in previous regularizations ([2], [3] and above) : we have a quantity which is log ultraviolet divergent but infrared finite, with $\Lambda$ playing the role of ultraviolet cut-off and $\frac{1}{\Lambda}$ the role of infrared cut-off.

We compute now easily the contributions from $\delta P^+$, by using the explicit parity symmetry of our formalism:

$$\delta P^+ = -\frac{e^2}{16\pi^2}\{\frac{p^2-m^2}{p^-}[ln\frac{\Lambda^2}{m^2}(2ln\frac{m^2}{\Lambda^2}-\frac{3}{2})-\int_0^{x_{max}} dx(\frac{2}{[x]}-2+x)ln(x-x(1-x)\frac{p^2}{m^2})]$$

$$-2\frac{p^2}{p^-}[\frac{3}{2}ln\frac{\Lambda^2}{m^2}-\int_0^{x_{max}} dx(2-x)ln(x-x(1-x)\frac{p^2}{m^2})]\} \tag{5.11}$$

Putting all of this together, we get

$$\delta M^2 = p^+(\delta P^-)_{p^2=m^2=p^+p^-} + p^-(\delta P^+)_{p^2=m^2=p^+p^-}$$

$$= \frac{3e^2}{8\pi^2}m^2 ln\frac{\Lambda^2}{m^2} \tag{5.12}$$

We can also obtain now the fermion wave-function renormalization as calculated in the previous work [2]. We get

$$Z_2^{-1} = 1-(\frac{\partial\delta P^-}{\partial p^-})_{p^2=m^2=p^+p^-} - (\frac{\partial\delta P^+}{\partial p^+})_{p^2=m^2=p^+p^-}$$

$$Z_2^{-1} = 1-\frac{e^2}{4\pi^2}\{ln\frac{\Lambda^2}{m^2}(3-2ln\frac{m^2}{\Lambda^2})\} \tag{5.13}$$

We look now at the one-loop quantum correction to the photon mass and wave-function . We proceed like we did above, and do the a global cut-off regularization.

In this case, we have different limits on $\kappa_\perp$ and $x$ than above:

$$\frac{\kappa_\perp^2 + m^2}{[x]} + \frac{\kappa_\perp^2 + m^2}{[1-x]} \leq \Lambda^2 \tag{5.14}$$

This means that there is a new maximum $\kappa_{max}$

$$k_{max}^2 = x(1-x)\Lambda^2 - m^2 \tag{5.15}$$

so that all $\kappa_\perp$ integrals are bounded above by this $\kappa_{max}$. This also means that $x$ is regularized differently

$$\frac{m^2}{\Lambda^2} = x_{min} \leq x \leq x_{max} = 1 - \frac{m^2}{\Lambda^2} \tag{5.16}$$

We get one contribution from the piece of the Hamiltonian $\mathcal{H}_{e^+e^-A}$ plus a contribution from the piece $\mathcal{H}_{e^+e^-AA}$:

$$\delta P^- = \delta P_1^- + \delta P_2^- \tag{5.17}$$

For a photon of energy $q$ we get the following from the iteration of $\mathcal{H}_{e^+e^-A}$ (the overall minus sign comes from the fermion loop):

$$\delta P_1^- = (-1)\frac{(-2e)^2}{q^+}\int \frac{d^2k_\perp}{16\pi^3} dk^+ \theta(k^+)\theta(q^+ - k^+) Tr\{\chi^\dagger \chi \frac{q_i}{q^+}\frac{q^i}{q^+}\chi^\dagger \chi \frac{1}{q^- - k^- - (q-k)^-}\} \tag{5.18}$$

which becomes, upon using $[x] = \frac{[k^+]}{q^+}$ and $\kappa_\perp = k_\perp - xq_\perp$

$$\delta P_1^- = -\frac{4e^2}{q^+}\int_0^{\kappa_{max}} \frac{d(\kappa^2)}{16\pi^2}\int_{x_{min}}^{x_{max}} dx\{\frac{(1-x)xq_\perp^2}{x(1-x)q^2 - \kappa^2 - m^2}\} \tag{5.19}$$

Upon integration, we get:

$$\delta P_1^- = \frac{e^2}{16\pi^2}\frac{q^+q^- - q^2}{q^+}[\frac{4}{6}ln\frac{\Lambda^2}{m^2} - \int_{x_{min}}^{x_{max}} dx x(1-x) ln(1 - x(1-x)\frac{q^2}{m^2})] \tag{5.20}$$

where the meaning of $\Lambda$ is as before.

For the second contribution, we have as follows after tying together $\mathcal{H}_{e^+e^-AA}$:

$$\delta P_2^- = e^2 \int \frac{d^2k_\perp}{16\pi^3} \frac{dk^+\theta(k^+)}{[k^+]} \frac{Tr\sigma_i\sigma^i}{[q^+ - k^+]} \tag{5.21}$$

which becomes, upon using $[x] = \frac{[k^+]}{q^+}$ and $\kappa_\perp = k_\perp - xq_\perp$

$$\delta P_2^- = \frac{e^2}{q^+} \int \frac{d^2k_\perp}{16\pi^2} \int_0^\infty \frac{dx}{[x]} \frac{2}{[1-x]} \tag{5.22}$$

Upon change of variables, we get:

$$\delta P_2^- = \frac{e^2}{q^+} \frac{1}{16\pi^2} \int_{x_{min}}^{x_{max}} dx \frac{k_{max}^2}{[x]} \tag{5.23}$$

This is just integrates to

$$\delta P_2^- = \frac{e^2}{16\pi^2} \{\Lambda^2(\frac{1}{2} - \frac{m^2}{\Lambda^2}) - m^2 ln(\frac{\Lambda^2}{m^2} - 1)\} \tag{5.24}$$

The total contribution to $\delta P^-$ is

$$\delta P^- = \frac{e^2}{24\pi^2} \frac{q^+q^- - q^2}{q^+} ln\frac{\Lambda^2}{m^2} + \frac{e^2}{16\pi^2} \{\Lambda^2(\frac{1}{2} - \frac{m^2}{\Lambda^2}) - m^2 ln(\frac{\Lambda^2}{m^2} - 1)\} \tag{5.25}$$

We get a similar result for $\delta P^+$ :

$$\delta P^+ = \frac{e^2}{24\pi^2} \frac{q^-q^+ - q^2}{q^-} ln\frac{\Lambda^2}{m^2} + \frac{e^2}{16\pi^2} \{\Lambda^2(\frac{1}{2} - \frac{m^2}{\Lambda^2}) - m^2 ln(\frac{\Lambda^2}{m^2} - 1)\} \tag{5.26}$$

The correction to the photon mass becomes then

$$\delta\mu^2 = q^+(\delta P^-)_{q^2=0=q^+q^-} + q^-(\delta P^+)_{q^2=0=q^+q^-}$$

$$\delta\mu^2 = \frac{e^2}{16\pi^2} \{\Lambda^2(1 - 2\frac{m^2}{\Lambda^2}) - 2m^2 ln(\frac{\Lambda^2}{m^2} - 1)\} \tag{5.27}$$

Notice now that we pick a quadratic divergence for the photon mass. It seems that the global cut-off spoils our infrared regularization, which protected us from this divergence in the transverse cut-off regularization.

We get now the photon wave-function renormalization by simply using the formula introduced before [2] :

$$Z_3^{-1} = 1 - (\frac{\partial \delta P^-}{\partial q^-})_{q^2=0} - (\frac{\partial \delta P^+}{\partial q^+})_{q^2=0}$$

$$Z_3^{-1} = 1 + \frac{e^2}{12\pi^2} ln \frac{\Lambda^2}{m^2} \tag{5.28}$$

This also agrees with the usual canonical results [7] and with the previous calculations.

## 6. Conclusions

We investigated several regularization schemes and were able to recover the covariant results in all cases which preserved the infrared regularization introduced by Harindranath and Zhang [6], which we have also used. It is surprising that the transverse cut-off regularization works. We can understand its success as due to the way different subgraphs, like $\delta P_1^-$ ,$\delta P_2^-$ and $\delta P_3^-$ for the fermion add up so as to cancel the worst infrared divergencies . Of the regularizations used, only global cut-off regularization spoils the photon mass correction, since in that case the infrared regularization, which protects us in the other cases from acquiring a quadratic ultraviolet divergence, is cancelled by the terms coming from the global cut-off, $\kappa_{max}$. But this should not be that surprising, since this cut-off treats $P^\mu$ differently than $A^\mu$, hence it spoils gauge invariance of the covariant derivative $P^\mu - iA^\mu$. For that reason, it is not that useful in calculations as are the other regularizations, with dimensional regularization being the easiest.

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