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Null-Surface Quantization of Gauge Field Theories without Gauge Constraints*

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ABSTRACT

In this paper we work out the perturbation theory and the renormalization procedure for Abelian gauge fields quantized following the method inspired by the work of Penrose. We compute the fermion mass renormalization as well as the photon mass renormalization to one-loop order and find complete agreement with the usual canonical quantization results. We obtain exactly zero for the photon mass renormalization. We use transverse dimensional regularization for these results.

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In previous papers[1 2 3 4 5 6] we introduced a quantization which treated x^+ and x^- on equal footing, which we dubbed Penrose quantization[7 8] . In the present paper we present results in the perturbation theory for fermion self-energy and photon self-energy to first order in the gauge coupling α. The interesting result is that we obtain identically zero for photon mass correction to one-loop, a first such result for quantization on null-surfaces.

The outline is as follows: we give a very brief derivation of the Hamiltonians P^- and P^+ for evolution along $x^+ = 0$ and $x^- = 0$ respectively. We then carry out the renormalization procedure to one-loop, following closely the method of Harindranth and Zhang.[9] . We do this for the fermion self-energy and the photon self-energy.

The Lagrangean of QED is :

$$\mathcal{L} = -\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\partial^\nu\overline{\psi}\gamma_\nu\psi - \overline{\psi}\frac{i}{2}\partial^\nu\gamma_\nu\psi - m\overline{\psi}\psi + \mathcal{L}_I \tag{1}$$

where $\mathcal{L}_I$ is the interaction part of the Lagrangean:

$$\mathcal{L}_I = -e\overline{\psi}\gamma^\nu\psi A_\nu \tag{2}$$

Previous work [2] ,[1] showed how to construct the Hamiltonian densities $\mathcal{H}$ and $\mathcal{K}$

$$\mathcal{H} = \{\frac{1}{2}(B^{-2} + (E^{-2})) + 2e\psi_+^\dagger A_+\psi_+ + 2e\psi_-^\dagger A_-\psi_- + \psi_+^\dagger\gamma_0\gamma_i\frac{i\partial_i}{2}\psi_- - e\psi_+^\dagger\gamma_0\gamma_i\frac{A^i}{2}\psi_-$$

$$+\psi_-^\dagger\gamma_0\gamma_i\frac{i\partial_i}{2}\psi_+ - e\psi_-^\dagger\gamma_0\gamma_i\frac{A^i}{2}\psi_+ + m\psi_+^\dagger\frac{\gamma^-}{4}\psi_- + m\psi_-^\dagger\frac{\gamma^+}{4}\psi_+\} \tag{3}$$

$$\mathcal{K} = \{\frac{1}{2}(B^{+2} + (E^{+2})) + 2e\psi_-^\dagger A_-\psi_- + 2e\psi_+^\dagger A_+\psi_+ + \psi_-^\dagger\gamma_0\gamma_i\frac{i\partial_i}{2}\psi_+ - e\psi_-^\dagger\gamma_0\gamma_i\frac{A^i}{2}\psi_+$$

$$+\psi_+^\dagger\gamma_0\gamma_i\frac{i\partial_i}{2}\psi_- - e\psi_+^\dagger\gamma_0\gamma_i\frac{A^i}{2}\psi_- + m\psi_-^\dagger\frac{\gamma^+}{4}\psi_+ + m\psi_+^\dagger\frac{\gamma^-}{4}\psi_-\} \tag{4}$$

where $B^- = B^+ = F^{12}$ and $E^- = -E^+ = -\frac{1}{2}F^{+-}$.

Going now to the quantum commutators, we got the following relations for fields at equal $x^+ = y^+$, the usual null-surface 'time' :

$$[A^i_P(x^+, x^-, x_\perp), A^j_P(y^+, y^-, y_\perp)]_{x^+=y^+} = \frac{-i}{4}\epsilon(x^- - y^-)\delta^2(x_\perp - y_\perp)\delta^{ij} \tag{5}$$

$$\left\{\psi_+(x^+, x^-, x_\perp), \psi_+(y^+, y^-, y_\perp)\right\}_{x^+=y^+} = \Lambda_+\delta(x^- - y^-)\delta^2(x_\perp - y_\perp) \tag{6}$$

So the physical (quantized) degrees of freedom on $x^+ = 0$ are A^i_P , ψ_+ and $\psi^\dagger_+$.

For fields at equal $x^- = y^-$, a new null-surface 'time', we got:

$$[A^i_M(x^+, x^-, x_\perp), A^j_M(y^+, y^-, y_\perp)]_{x^-=y^-} = \frac{-i}{4}\epsilon(x^- - y^-)\delta^2(x_\perp - y_\perp)\delta^{ij} \tag{7}$$

$$\left\{\psi_-(x^+, x^-, x_\perp), \psi_-(y^+, y^-, y_\perp)\right\}_{x^-=y^-} = \Lambda_-\delta(x^+ - y^+)\delta^2(x_\perp - y_\perp) \tag{8}$$

Here, the physical (quantized) degrees of freedom on $x^- = 0$ are A^i_M , ψ_- and $\psi^\dagger_-$. Note that A^+ and A^- do not enter in the list of physical (quantized) degrees of freedom.

Rather, they are determined from requiring that $\mathcal{M}$ is zero on the initializing surfaces. The solutions were found to be [2] :

$$A^+_P = 0 \tag{9}$$

$$A^-_P = -\frac{2i\partial_i}{(i\partial^+)}A^i_P + \frac{4}{(i\partial^+)^2}e\psi^\dagger_+\psi_+ \tag{10}$$

on the surface $x^+ = 0$ and

$$A^-_M = 0 \tag{11}$$

$$A^+_M = -\frac{2i\partial_i}{(i\partial^-)}A^i_M + \frac{4}{(i\partial^-)^2}e\psi^\dagger_-\psi_- \tag{12}$$

As in the previous papers [4], [2] ,[1] the fields are defined on the initializing surfaces $x^+ = 0$, $x^- = 0$, and $x^+ = x^- = 0$ as follows: $A_i(x^+, x^-, x_\perp) = A_{iP}(x^-, x_\perp) + A_{iM}(x^+, x_\perp) - B_i(x_\perp)$,while the fermion field is defined to be $\psi(x^+, x^-, x_\perp) = \psi_-(x^-, x_\perp) + \psi_+(x^+, x_\perp)$. The field B_i is just the gauge field along the hypersurface $x^+ = x^- = 0$: $A_{iP}(x^- = 0, x_\perp) =$

$A_{iM}(x^+ = 0, x_\perp) = B_i(x_\perp)$. We've added B_i so that along that hypersurface we don't double count the field. Notice also that now along $x^+ = 0$, we have $A_i(x^+ = 0, x^-, x_\perp) = A_{iP}(x^-, x_\perp)$ and along $x^- = 0$ we like-wise have $A_i(x^+, x^- = 0, x_\perp) = A_{iM}(x^+, x_\perp)$ namely, that the total field A_i reduces to A_{iP} or A_{iM} along these surfaces, as it should.

Let us study now the Hamiltonian P^-. It is constructed out of the Hamiltonian density $\mathcal{H}$. But first, we must express all the fields in terms of the fields defined on $x^+ = 0$. We have

$$\mathcal{H} = \{\frac{1}{2}(B_P^{-2} + E_P^{-2}) + \psi_+^\dagger \psi_+ e A_P^- + \psi_+^\dagger \gamma_0 \gamma_i \frac{i\partial_i}{2}\psi_- - e\psi_+^\dagger \gamma_0 \gamma_i \frac{A_P^i}{2}\psi_-$$

$$+\psi_-^\dagger \gamma_0 \gamma_i \frac{i\partial_i}{2}\psi_+ - e\psi_-^\dagger \gamma_0 \gamma_i \frac{A_P^i}{2}\psi_+ + m\psi_+^\dagger \frac{\gamma^-}{4}\psi_- + m\psi_-^\dagger \frac{\gamma^+}{4}\psi_+\} \tag{13}$$

where $B_P^- = (\partial^1 A_P^2 - \partial^2 A_P^1)$ and $E_P^- = -\frac{1}{2}\partial^+ A_P^-$. We follow the notation of [9] and observe that the fermionic fields are $\psi_+^\dagger = (\xi^\dagger, 0)$ and $\psi_-^\dagger = (0, \eta^\dagger)$.

Substituting this in the equation [(13)] we get

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I \tag{14}$$

where

$$\mathcal{H}_0 = \frac{1}{2}A_P^i(\partial_i^2)A_P^i + \frac{1}{2}\xi^\dagger \frac{(-\partial_i^2 + m^2)}{i\partial^+}\xi \tag{15}$$

while the interaction Hamiltonian density is much more complicated:

$$\mathcal{H}_I = \mathcal{H}_{e^+e^-A} + \mathcal{H}_{e^+e^-AA} + \mathcal{H}_{e^+e^-e^+e^-} \tag{16}$$

These pieces are as follows:

$$\mathcal{H}_{e^+e^-A} = -e\xi^\dagger \frac{i\partial_k}{i\partial^+}A_P^k \xi + \frac{e}{2}\xi^\dagger \sigma_i A_P^i \frac{1}{i\partial^+}(\sigma_j i\partial_j + im)\xi + \frac{e}{2}\xi^\dagger \frac{1}{i\partial^+}(\sigma_j i\partial_j - im)\sigma_i A_P^i \xi$$

$$-e\xi^\dagger i\partial_k A_P^k \frac{1}{i\partial^+}\xi + \frac{e}{2}\xi^\dagger \sigma_i (\sigma_j i\partial_j + im)\frac{1}{i\partial^+}A_P^i \xi + \frac{e}{2}\xi^\dagger \sigma_i A_P^i (\sigma_j i\partial_j - im)\frac{1}{i\partial^+}\xi$$

$$-2e\xi^\dagger \xi \frac{1}{i\partial^+}i\partial_i A_P^i \tag{17}$$

$$\mathcal{H}_{e^+e^-AA} = e^2 \xi^\dagger \sigma_i A^i_P \frac{1}{i\partial^+} \sigma_j A^j_P \xi \tag{18}$$

$$\mathcal{H}_{e^+e^-e^+e^-} = 4e^2 \xi^\dagger \xi \frac{1}{(i\partial^+)^2} \xi^\dagger \xi - 2e^2 \frac{1}{(i\partial^+)} \xi^\dagger \xi \frac{1}{(i\partial^+)} \xi^\dagger \xi \tag{19}$$

We follow now the approach of Bjorken, Kogut and Soper [10] as well as the approach of Harindranath and Zhang [9] to obtain a perturbative theory of QED, ordered in x^+ as well as in x^-, since here both of these coordinates are 'times.' The details are given in an upcoming paper [1].

We apply the perturbation theory developed in earlier work [1] to calculating the fermion mass renormalization and wave-function renormalization to one-loop.

Since both P^+ and P^- are dynamical, we get contributions from both of them. First, let us compute the contributions from P^-, denoted by δP^-, for an electron with momentum p :

$$\delta P^- = \delta P^-_1 + \delta P^-_2 + \delta P^-_3 \tag{20}$$

where δP^-_1 is due to the emission and reabsorbtion of a virtual photon ,coming from the iteration of $\mathcal{H}_{e^+e^-A}$; the second piece comes from the four lepton piece of the Hamiltonian, $\mathcal{H}_{e^+e^-e^+e^-}$, where two of the fermions have been 'tied' together; the last piece comes from the two-photon piece of the Hamiltonian, $\mathcal{H}_{e^+e^-AA}$, where the photon lines have been 'tied' together

The first contribution becomes, after performing transverse dimensional regularization :

$$\delta P^-_1 = -\frac{e^2}{16\pi^2} \{ \frac{p^2 - m^2}{p^+} [\frac{1}{\epsilon_t} (2ln\frac{p^+}{\lambda} - \frac{3}{2}) - \int_0^1 dx (\frac{2}{[x]} - 2 + x) ln(x\frac{m^2}{\mu^2} - x(1-x)\frac{p^2}{\mu^2})]$$

$$-2\frac{p^2}{p^+}[\frac{3}{2\epsilon_t} - \int_0^1 dx(2-x) ln(x\frac{m^2}{\mu^2} - x(1-x)\frac{p^2}{\mu^2})]\} \tag{21}$$

where $\epsilon_t = 1 - \frac{d_t}{2}$ (so that $\frac{1}{\epsilon_t}$ is equivalent to $ln\frac{\Lambda_\perp^2}{m^2}$ in ultraviolet cut-off regularization), and

λ is the infra-red regularization. The second and third contributions are identically zero. The total contribution to δP^- is just δP_1^-.

We compute now easily the contributions from δP^+, by using the explicit parity symmetry of our formalism:

$$\delta P^+ = -\frac{e^2}{16\pi^2}\{\frac{p^2-m^2}{p^-}[\frac{1}{\epsilon_t}(2ln\frac{p^-}{\lambda}-\frac{3}{2})-\int_0^1 dx(\frac{2}{[x]}-2+x)ln(x\frac{m^2}{\mu^2}-x(1-x)\frac{p^2}{\mu^2})]$$

$$-2\frac{p^2}{p^-}[\frac{3}{2\epsilon_t}-\int_0^1 dx(2-x)ln(x\frac{m^2}{\mu^2}-x(1-x)\frac{p^2}{\mu^2})]\} \tag{22}$$

Putting all of this together, we get

$$\delta M^2 = p^+(\delta P^-)_{p^2=p^+p^-=m^2} + p^-(\delta P^+)_{p^2=p^+p^-=m^2}$$

$$= \frac{3e^2}{8\pi^2}m^2\frac{1}{\epsilon_t} \tag{23}$$

which is identical to the usual canonical quantization result[11] . The key is that we get the same singularity structure. Note the feature that infrared singularities cancel and ultraviolet singularities are only logarithmic, like in the canonical treatment.

We look now at the one-loop quantum correction to the photon mass. We proceed like we did above and get several contributions from the remaining pieces of the Hamiltonian, $\mathcal{H}_{e^+e^-A}$ and $\mathcal{H}_{e^+e^-AA}$:

$$\delta P^- = \delta P_1^- + \delta P_2^- \tag{24}$$

For a photon of energy q the first contribution is from the iteration of $\mathcal{H}_{e^+e^-A}$ which becomes:

$$\delta P_1^- = \frac{e^2}{24\pi^2}\frac{q^+q^- - q^2}{q^+}\frac{1}{\epsilon_t} \tag{25}$$

where the meaning of ϵ_t and λ is as before.

For the second contribution, we "tie" the fermion lines in $\mathcal{H}_{e^+e^-AA}$ and obtain $\delta P_2^- = 0$. The total contribution to δP^- is δP_1^-.

We get a similar result for δP^+'s :

$$\delta P^+ = \frac{e^2}{24\pi^2}\frac{q^-q^+ - q^2}{q^-}\frac{1}{\epsilon_t} \tag{26}$$

The correction to the photon mass becomes then

$$\delta\mu^2 = q^+(\delta P^-)_{q^2=q^+q^-=0} + q^-(\delta P^+)_{q^2=q^+q^-=0}$$

$$\delta\mu^2 = 0 \tag{27}$$

Note that the one-loop correction to the photon mass is exactly zero!

1. Conclusions

This new method of quantizing fields on null-surfaces seem to give consistent results in perturbation theory, applied to QED. It seems that the singularity structure is simplified, so that to one-loop the infrared and ultraviolet singularities are logarithmic only, as in canonical quantization. We also obtain that the photon mass stays zero to one-loop. This is the first null-surface calculation which gives a zero photon mass renormalization (to one loop). It is reassuring that we get this result, since we quantized only the physical degrees of freedom. It seems that using null-surface Penrose quantization we obtain physical answers more directly than by other methods.

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