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Null-Surfaces Quantization of Quantum ChromoDynamics in the Heavy Quark Limit: Leading $\frac{1}{m_Q}$ Calculation*

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ABSTRACT

In this paper we extend null-surfaces quantization to local non-Abelian gauge fields in the heavy quark limit. We quantize this theory using a method proposed originally by Faddeev and Jackiw . As in previous work, we find that quantizing along both x^+ and x^- , the gauge theory does not require constraints. We compute the effective Lagrangean and Hamiltonians to leading order in $\frac{1}{m_Q}$.

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1. Introduction

In previous papers [1] [2] [3] [4] [5] [6] we introduced a quantization which treated x^+ and x^- on equal footing, dubbed null-surfaces quantization. (See also [7] [8] for discussions on treating x^+ and x^- equally.) In this paper we extend our previous study to the non-Abelian ($SU(N)$) case in the presence of heavy quarks [9] [10] [11] [12] . We obtain some results similar to the Abelian case : no extra conditions are needed between the quantizing degrees of freedom (the initial data). Furthermore, there are simplifications in this formalism, in that we don't have to solve difficult constraint equations. While in provious work [1] we had started with two flavors of quarks, one very heavy, one light, here we start with one type of heavy quark and then apply the heavy flavor symmetry. We obtain then an effective Lagrangean (and Hamiltonians) in these new degrees of freedom, so that we show at the end how to compare the results obtained in the present formalism with previous work 1].

We are able to identify the light and heavy degrees of freedom with ψ_- and ψ_+ respectively, on $x^+ = 0$. We are also able to construct the effective Lagrangean and the corresponding Hamiltonians to leading order in the heavy quark mass $\frac{1}{m_Q}$. We are then able to generate the zeroth order and first order Hamiltonians.

We first review the reduced phase space method of Faddeev and Jackiw. We then discuss the quantization of the fields. With these issues resolved, we construct the effective Hamiltonians to leading order in $\frac{1}{m_Q}$. The presence of the natural perturbative parameter $\frac{1}{m_Q}$ allows us to develop a perturbative expansion in this parameter. We review then the perturbation theory we had developed for QED and QCD in previous papers [5], [1] and apply this to our present formalism. We then make a few remarks regarding an application of this method to heavy-light currents. In particular, we show how to compare the calculations in the QCD formulation, presented elsewhere [1], with those calculations stemming from the present formalism.

2. Reduced Phase Space Quantization of QCD

We follow the approach of Neubert [10] and start with the following action to study heavy quark QCD :

$$\mathcal{L} = -\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\partial^{\nu}\overline{Q}\gamma_{\nu}Q - \overline{Q}\frac{i}{2}\partial^{\nu}\gamma_{\nu}Q - m_Q\overline{Q}Q + \mathcal{L}_I \tag{2.1}$$

where $\mathcal{L}_I$ is the interaction part of the Lagrangean:

$$\mathcal{L}_I = +g\overline{Q}\not{A}Q \tag{2.2}$$

In this case, we have $F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\mu}A^{\nu} - ig[A^{\mu}, A^{\nu}]$, $A^{\mu} = \sum_a A^{\mu}_a T^a$ is the gluon field, T^a are the generators for the color $SU(3)$ group, with $[T^a, T^b] = if^{abc}T^c$, f^{abc} being the $SU(3)$ color structure functions. We also have $Tr(T^aT^b) = \frac{1}{2}\delta^{ab}$. The 'velocity' v is the part of the heavy quark momentum proportional to the heavy quark mass m_Q [10] :

$$p_{\mu} = v_{\mu}m_Q + k_{\mu} \tag{2.3}$$

and this v is normalized to one $v^2 = v^+v^- - v_iv^i = 1$. We define the fermion field Q as usual form [10] :

$$Q(x) = e^{-im_Q v.x}[h_v(x) + H_v(x)] \tag{2.4}$$

so that the 'large' and 'small' components h_v and H_v are defined as

$$h_v = e^{im_Q v.x} P_+ Q(x) \tag{2.5}$$

$$H_v = e^{im_Q v.x} P_- Q(x) \tag{2.6}$$

where the projection operator $P_\pm = \frac{1}{2}(1 \pm \not{v})$. We also have the identities $\not{v} h_v = h_v$, $\not{v} H_v = -H_v$ and $\gamma_3 h_v = -H_v$, $\gamma_3 H_v = -h_v$. In the rest frame, h_v creates a heavy quark with velocity v , while H_v creates a heavy anti-quark with velocity v. If we want to describe a heavy antiquark, we change the sign of the velocity, and introduce the definitions

$$h_v^- = e^{-im_Q v.x} P_- Q(x) \tag{2.7}$$

$$H_v^- = e^{-im_Q v.x} P_+ Q(x) \tag{2.8}$$

so that the field Q becomes

$$Q(x) = e^{im_Q v.x} [h_v^-(x) + H_v^-(x)] \tag{2.9}$$

The classical equations of motion are

$$-iv.D h_v = i\not{D}_\perp H_v \tag{2.10}$$

$$(iv.D + 2m_Q) H_v = i\not{D}_\perp h_v \tag{2.11}$$

where $D_\perp^\mu = D^\mu - v^\mu v.D$ and $iD^\mu = i\partial^\mu + gA_a^\mu T^a$. The Lagrangean becomes then

[10]

$$\mathcal{L}_{eff} = \overline{h}_v iv.Dh_v - \overline{H}_v(iv.D + 2m_Q)H_v + \overline{h}_v i\not{D}_\perp H_v + \overline{H}_v i\not{D}_\perp h_v \tag{2.12}$$

To lowest order in $\frac{1}{m_Q}$, this gives the following for the effective Lagrangean [10]

$$\mathcal{L}_{eff} = \overline{h}_v iv.Dh_v + \frac{1}{2m_Q}\overline{h}_v(iD_\perp)^2 h_v + \frac{g}{4m_Q}\overline{h}_v\sigma_{\mu\nu}F^{\mu\nu}h_v \tag{2.13}$$

Notice now several interesting features: in usual light-cone methods,we use the projections $\Lambda_\pm = \frac{1}{2}\gamma^0\gamma^\pm$ so that $\psi_\pm = \Lambda_\pm\psi$. There are some interesting results which allow us to identify h_v with the usual ψ_+! Namely,

$$\Lambda_\pm\psi = [\frac{1}{2}(1 \pm \gamma_3)P_+ + \frac{1}{2}(1 \mp \gamma_3)P_-]Q(x) \tag{2.14}$$

We get the results that $\psi_+ = h_v$, $\psi_- = H_v$, or $\psi_- = h_v^-$, $\psi_+ = H_v^-$. We also use the fact that $P_+\gamma^\mu P_+ = P_+v^\mu P_+$ and write $\psi_+^\dagger v^0$ for $h_v^\dagger\gamma^0$, for instance. We re-write the effective Lagrangean as follows

$$\mathcal{L}_{eff} = \psi_+^\dagger v^0 iv.D\psi_+ + \frac{1}{2m_Q}\psi_+ \dagger\, v^0(iD_\perp)^2\psi_+ + \frac{g}{4m_Q}\psi_+^\dagger v^0\sigma_{\mu\nu}F^{\mu\nu}\psi_+$$

$$+\psi_-^\dagger v^0 iv.D\psi_- + \frac{1}{2m_Q}\psi_- \dagger\, v^0(iD_\perp)^2\psi_- + \frac{g}{4m_Q}\psi_-^\dagger v^0\sigma_{\mu\nu}F^{\mu\nu}\psi_- \tag{2.15}$$

To obtain the Hamiltonians for evolution along x^+ and x^-, we follow the approach of previous papers [10], [9], [3], [4], [5] and [6] , so we write $\mathcal{L}_{eff}$ out explicitly :

$$\mathcal{L}_{eff} d^4x = \{ -\frac{1}{2} F_{ija} F^{ija} - F_{+-a} F^{+-a} - F_{+ia} F^{+ia} - F_{-ia} F^{-ia}$$

$$+ g\psi_+^\dagger v^0 (\frac{A^- v^+}{2} + \frac{A^+ v^-}{2} - A^i v_i)\psi_+ + g\psi_-^\dagger v^0 (\frac{A^- v^+}{2} + \frac{A^+ v^-}{2} - A^i v_i)\psi_-$$

$$+ \psi_+^\dagger \frac{v^0}{2} (\frac{v^+ i\partial^-}{2} + \frac{v^- i\partial^+}{2} - i v_i \partial^i)\psi_+ - [(\frac{v^+ i\partial^-}{2} + \frac{v^- i\partial^+}{2} - i v_i \partial^i)\psi_+^\dagger] \frac{v^0}{2} \psi_+$$

$$+ \psi_-^\dagger \frac{v^0}{2} (\frac{v^+ i\partial^-}{2} + \frac{v^- i\partial^+}{2} - i v_i \partial^i)\psi_- - [(\frac{v^+ i\partial^-}{2} + \frac{v^- i\partial^+}{2} - i v_i \partial^i)\psi_-^\dagger] \frac{v^0}{2} \psi_-$$

$$+ \frac{1}{2m_Q} \psi_+^\dagger v^0 (iD_\perp)^2 \psi_+ + \frac{1}{2m_Q} \psi_-^\dagger v^0 (iD_\perp)^2 \psi_-$$

$$+ \frac{g}{4m_Q} \psi_+^\dagger \frac{iv^0}{2} (\gamma_i \gamma_j F^{ij} + \gamma_+ \gamma_i F^{+i})\psi_+ + \frac{g}{4m_Q} \psi_-^\dagger \frac{iv^0}{2} (\gamma_i \gamma_j F^{ij} + \gamma_- \gamma_i F^{-i})\psi_- \} \frac{dx^- dx^+ d^2 x_\perp}{2} \tag{2.16}$$

Note also that $\partial_\nu = \frac{\partial}{\partial x^\nu}$ so that $\partial^- = 2\partial_+ = 2\frac{\partial}{\partial x^+}$ and $\partial^+ = 2\partial_- = 2\frac{\partial}{\partial x^-}$. The corresponding conjugate momenta for x^+-derivatives are

$$\pi^i_{Aa} = \frac{\partial \mathcal{L}}{\partial(\partial_+ A_{ia})} = -F_a^{+i} \tag{2.17}$$

$$\pi_\psi(x) = \frac{\partial \mathcal{L}}{\partial(\partial_+ \psi_+)} = \frac{i}{2} \psi_+^\dagger v^+ v^0 \tag{2.18}$$

$$\pi_{\psi^\dagger}(x) = \frac{\partial \mathcal{L}}{\partial(\partial) + \psi_+^\dagger)} = -\frac{i}{2} \psi_+ v^+ v^0 \tag{2.19}$$

For the momenta corresponding to x^--derivatives we get similar forms:

$$\rho^i_{Aa} = \frac{\partial \mathcal{L}}{\partial(\partial_- A_{ia})} = -F_a^{-i} \tag{2.20}$$

$$\rho_\psi(x) = \frac{\partial \mathcal{L}}{\partial(\partial_- \psi_-)} = \frac{i}{2}\psi_-^\dagger v^- v^0 \tag{2.21}$$

$$\rho_{\psi^\dagger}(x) = \frac{\partial \mathcal{L}}{\partial(\partial_- \psi_-^\dagger)} = -\frac{i}{2}\psi_- v^- v^0 \tag{2.22}$$

Since neither v^+ nor v^- are zero, we can rescale the fermionic fields as follows : $\psi_+ \to \sqrt{v^- v^0}\psi_+$ and $\psi_- \to \sqrt{v^- v^0}\psi_-$. Then the conjugate momenta become

$$\pi_\psi(x) = \frac{i}{2}\psi_+^\dagger \frac{v^+}{v^-},\ \pi_{\psi^\dagger}(x) = -\frac{i}{2}\psi_+ \frac{v^+}{v^-} \tag{2.23}$$

$$\rho_\psi(x) = \frac{i}{2}\psi_-^\dagger \frac{v^-}{v^+},\ \rho_{\psi^\dagger}(x) = -\frac{i}{2}\psi_- \frac{v^-}{v^+} \tag{2.24}$$

We rewrite $\mathcal{L}d^4x$ in the following way

$$\mathcal{L}d^4x = \frac{1}{2}\{\pi^i_{Aa}\partial_+ A_{ia} + \pi^i_{Aa}\partial_+ A_{ia} + \rho^i_{Aa}\partial_- A_{ia} + \rho^i_{Aa}\partial_- A_{ia}$$

$$+\pi_\psi \partial_+ \psi_+ + \pi_\psi \partial_+ \psi_+ + (\partial_+ \psi_+^\dagger)\pi_{\psi^\dagger} + (\partial_+ \psi_+^\dagger)\pi_{\psi^\dagger}$$

$$+\rho_\psi \partial_- \psi_- + \rho_\psi \partial_- \psi_- + (\partial_- \psi_-^\dagger)\rho_{\psi^\dagger} + (\partial_- \psi_-^\dagger)\rho_{\psi^\dagger}\}d^4x$$

$$-\mathcal{H}dx^4 - \mathcal{K}dx^4 + \mathcal{M}d^4x \tag{2.25}$$

The meaning of these terms is as in the previous papers [3], [4], [5] and [6] , so we don't go into details here. The Hamiltonian densities $\mathcal{H}$ and $\mathcal{K}$ are (to order $\frac{1}{m_Q}$):

$$\mathcal{H} = \{\frac{1}{2}(B_a^{-2} + E_a^{-2}) - \frac{g}{2v^-}\psi_+^\dagger(A^- v^+ + A^+ v^- - A^i v_i)\psi_+ - \frac{1}{2m_Q v^-}\psi_+^\dagger(iD_\perp)^2\psi_+$$

$$-\psi_+^\dagger(\frac{i\partial^+}{4} - \frac{iv_i\partial^i}{2v^-})\psi_+ + [(\frac{i\partial^+}{4} - \frac{iv_i\partial^i}{2v^-})\psi_+^\dagger]\psi_+$$

$$-\frac{g}{4m_Qv^-}\psi_+^\dagger\frac{i}{2}(\gamma_i\gamma_jF^{ij}+\gamma_+\gamma_iF^{+i})\psi_+\} \tag{2.26}$$

$$\mathcal{K}=\{\frac{1}{2}(B_a^{+2}+E_a^{+2})-\frac{g}{2v^+}\psi_-^\dagger(A^+v^-+A^-v^+-A^iv_i)\psi_- -\frac{1}{2m_Qv^+}\psi_-^\dagger(iD_\perp)^2\psi_-$$

$$-\psi_-^\dagger(\frac{i\partial^-}{4}-\frac{iv_i\partial^i}{2v^+})\psi_- +[(\frac{i\partial^-}{4}-\frac{iv_i\partial^i}{2v^+})\psi_-^\dagger]\psi_-$$

$$-\frac{g}{4m_Qv^+}\psi_-^\dagger\frac{i}{2}(\gamma_i\gamma_jF^{ij}+\gamma_-\gamma_iF^{-i})\psi_-\} \tag{2.27}$$

where $B_a^- = B_a^+ = F_a^{12}$ and $E_a^- = -E_a^+ = -\frac{1}{2}F_a^{+-}$. We have dropped the terms $\frac{g}{2v^+}\psi_-^\dagger(A^-v^++A^+v^--A^iv_i)\psi_-$ (and $\frac{g}{2v^-}\psi_+^\dagger(A^-v^++A^+v^--A^iv_i)\psi_+$ respectively) in $\mathcal{H}$ ($\mathcal{K}$) since they are of order $\frac{1}{m_Q^2}$. For the 'constraints' we get whatever is left over

$$\mathcal{M}=\{-\partial_iA_{+a}F^{+ia}-\partial_iA_{-a}F^{-ia}-gA_{+a}A_{ib}f^{abc}F^{i+a}-gA_{-a}A_{ib}f^{abc}F^{i-a}$$

$$+F_{+-a}F^{+-a}-\frac{g}{2v^-}\psi_+^\dagger(A^-v^++A^+v^-)\psi_+ -\frac{g}{2v^+}\psi_-^\dagger(A^-v^++A^+v^-)\psi_-\} \tag{2.28}$$

We can rewrite this as follows (up to total derivatives) :

$$\mathcal{M}=A_{+a}C_\pi^a+A_{-a}C_\rho^a \tag{2.29}$$

and the 'constraints' C_π^a and C_ρ^a are (to order $\frac{1}{m_Q}$)

$$C_\pi^a=-\partial_-F^{-+a}-\partial_iF^{i+a}+gf^{abc}A_{ib}F_c^{i+}-g\frac{v^+}{v^-}\psi_+^\dagger T^a\psi_+ \tag{2.30}$$

$$C_\rho^a=-\partial_+F^{+-a}-\partial_iF^{i-a}+gf^{abc}A_{ib}F_c^{i-}-g\frac{v^-}{v^+}\psi_-^\dagger T^a\psi_- \tag{2.31}$$

We want now $\mathcal{M}=0$ so that we can identify unambiguously $\mathcal{H}$ and $\mathcal{K}$. We'll explore how to do this in the next section.

Let us write the $\mathcal{L}_{eff} dx^4$ with the explicit momenta dependence (up to total derivatives which we can discard [13] [14] even for fermions [2]) so as to make the resulting commutation relation clear :

$$\mathcal{L}_{eff} d^4x = \frac{1}{2}\{\pi^i_{Aa} dA^a_i - A_{ia} d\pi^{ia}_A + \pi_\psi d\psi_+ - d\pi_\psi \psi_+ + d\psi^\dagger_+ \pi_{\psi^\dagger} - \psi^\dagger_+ d\pi_{\psi^\dagger}\}\frac{dx^- dx_\perp}{2}$$

$$+\frac{1}{2}\{\rho^i_{Aa} dA^a_i - A_{ia} d\rho^{ia}_A + \rho_\psi d\psi_- - d\rho_\psi \psi_- + d\psi^\dagger_- \rho_{\psi^\dagger} - \psi^\dagger_- d\rho_{\psi^\dagger}\}\frac{dx^+ dx_\perp}{2}$$

$$-\mathcal{H} dx^4 - \mathcal{K} dx^4 \qquad (2.32)$$

We see now that we have two types of evolutions, one along x^+, for which the first term in equation (2.32) gives the commutation relations along surfaces $x^+ = y^+$ according to the form:

$$[\xi^i, \xi^i] = \Gamma^{-1}_{ij} \quad i, j = 1, ..8 \qquad (2.33)$$

with

$$\xi^1 = \pi^1_A, \xi^2 = \pi^2_A, \xi^3 = \pi_\psi, \xi^4 = \pi_{\psi^\dagger}, \xi^5 = A^1, \xi^6 = A^2, \xi^7 = \psi_+, \xi^8 = \psi^\dagger_+ \qquad (2.34)$$

and

$$\Gamma_{15} = \Gamma_{26} = \Gamma_{37} = \Gamma_{48} = 1 = -\Gamma_{51} = -\Gamma_{62} = -\Gamma_{73} = -\Gamma_{84} \qquad (2.35)$$

and all the other Γ's are 0 . The second term in equation (2.32) gives the commutation relations along surfaces $x^- = y^-$ according to the form :

$$[\eta^i, \eta^j] = \Delta^{-1}_{ij} \quad i, j = 1, ..8 \qquad (2.36)$$

with

$$\eta^1 = \rho^1_A, \eta^2 = \rho^2_A, \eta^3 = \pi_\psi, \eta^4 = \pi_{\psi^\dagger}, \eta^5 = A^1, \eta^6 = A^2, \eta^7 = \psi_-, \eta^8 = \psi^\dagger_- \quad (2.37)$$

and

$$\Delta_{15} = \Delta_{26} = \Delta_{37} = \Delta_{48} = 1 = -\Delta_{51} = -\Delta_{62} = -\Delta_{73} = -\Delta_{84} \quad (2.38)$$

and all the other Δ's are 0 . Going now to the quantum commutators, we get the following relations for fields at equal $x^+ = y^+$, the usual front-form 'time' :

$$[A^i_{Pa}(x^+, x^-, x_\perp), \pi^j_{Ab}(y^+, y^-, y_\perp)]_{x^+=y^+} = i\delta(x^- - y^-)\delta^2(x_\perp - y_\perp)\delta^{ij}\delta_{ab} \quad (2.39)$$

$$\left\{\psi_+(x^+, x^-, x_\perp), \pi_\psi(y^+, y^-, y_\perp)\right\}_{x^+=y^+} = +\frac{i}{2}\Lambda_+\frac{v^+}{v^-}\delta(x^- - y^-)\delta^2(x_\perp - y_\perp) \quad (2.40)$$

$$\left\{\psi^\dagger_+(x^+, x^-, x_\perp), \pi_{\psi^\dagger}(y^+, y^-, y_\perp)\right\}_{x^+=y^+} = -\frac{i}{2}\Lambda_+\frac{v^+}{v^-}\delta(x^- - y^-)\delta^2(x_\perp - y_\perp) \quad (2.41)$$

The physical (quantized) degrees of freedom on $x^+ = 0$ are A^i_{Pa} , ψ_+ and $\psi^\dagger_+$.

For fields at equal $x^- = y^-$, a new front-form 'time', we get:

$$[A^i_{Ma}(x^+, x^-, x_\perp), \rho^j_{Ab}(y^+, y^-, y_\perp)]_{x^-=y^-} = i\delta(x^+ - y^+)\delta^2(x_\perp - y_\perp)\delta^{ij}\delta_{ab} \quad (2.42)$$

$$\left\{\psi_-(x^+, x^-, x_\perp), \rho_\psi(y^+, y^-, y_\perp)\right\}_{x^-=y^-} = +\frac{i}{2}\Lambda_-\frac{v^-}{v^+}\delta(x^+ - y^+)\delta^2(x_\perp - y_\perp) \quad (2.43)$$

$$\left\{\psi^\dagger_-(x^+, x^-, x_\perp), \rho_{\psi^\dagger}(y^+, y^-, y_\perp)\right\}_{x^-=y^-} = -\frac{i}{2}\Lambda_-\frac{v^-}{v^+}\delta(x^+ - y^+)\delta^2(x_\perp - y_\perp) \quad (2.44)$$

Here, the physical (quantized) degrees of freedom on $x^- = 0$ are A^i_{Ma} , ψ_- and $\psi^\dagger_-$. Note that A^+_a and A^-_a do not enter in the list of physical (quantized) degrees of freedom.

The equations of motions are now like in Faddeev and Jackiw [13] and as in out previous papers [5], [4]:

$$\Gamma_{ij}\partial_+\xi^j = \frac{\partial\mathcal{H}}{\partial\xi^i} \tag{2.45}$$

for the x^+ variation, and

$$\Delta_{ij}\partial_-\eta^j = \frac{\partial\mathcal{K}}{\partial\eta^i} \tag{2.46}$$

for the x^- variation . For $i = 7$ and $j = 3$ we recover the equation of motion for π_ψ

$$\frac{v^+}{v^-}i\partial^-\psi_+^\dagger = +[\frac{i\partial^+}{2} - \frac{iv_i\partial^i}{v^-}]\psi_+^\dagger - \frac{g}{v^-}\psi_+^\dagger(A^-v^+ + A^+v^- - A^iv_i)$$

$$-\frac{1}{m_Qv^-}\psi_+^\dagger(iD_\perp)^2 - \frac{g}{4m_Qv^-}\psi_+^\dagger i(\gamma_i\gamma_jF^{ij} + \gamma_+\gamma_iF^{+i}) \tag{2.47}$$

We get similar results from (2.46) for $\rho_{\psi^\dagger}$:

$$\frac{v^-}{v^+}i\partial^+\psi_- = -[\frac{i\partial^-}{2} - \frac{iv_i\partial^i}{v^+}]\psi_- - \frac{g}{v^+}(A^-v^+ + A^+v^- - A^iv_i)\psi_-$$

$$-\frac{1}{m_Qv^+}(iD_\perp)^2\psi_- - \frac{g}{4m_Qv^+}i(\gamma_i\gamma_jF^{ij} + \gamma_-\gamma_iF^{-i})\psi_- \tag{2.48}$$

3. Quantization of the Fields

The quantization of the fields $A^i_{P,Mb}$ $\psi_\pm$ goes through as in the previous work [5], [4], [3]. It is only in the components A^+_a and A^-_a that any surprises might arise. For completeness, we write out the resulting fields anyway.

On $x^+ = 0$ we have the following definitions for the 'plus-moving' fields:

$$A_{iPb}(x^-, x_\perp) = \int \frac{d^2k_\perp}{(2\pi)^3}\frac{dk^+}{2k^+}\sum_\lambda \left\{\epsilon_i^\lambda a_b(k^+, k_\perp, \lambda)e^{-ik.x} + \epsilon_i^{*\lambda} a_b^\dagger(k^+, k_\perp, \lambda)e^{+ik.x}\right\} \tag{3.1}$$

$$\psi_+(x^-, x_\perp) = \int \frac{d^2k_\perp}{(2\pi)^3}\frac{dk^+}{2k^+}\sum_\lambda \left\{b(k^+, k_\perp)u_+(k^+, k_\perp, \lambda)e^{-ik.x}\right.$$

$$\left. + d^\dagger(k^+, k_\perp)v_+(k^+, k_\perp, \lambda)e^{+ik.x}\right\} \tag{3.2}$$

In this case, $ik.x = \frac{i}{2}k^+x^- - ik_\perp.x_\perp$ and the polarization vectors are $\epsilon_i^{+1} = \frac{1}{\sqrt{2}}(1, i); \epsilon_i^{-1} = \frac{1}{\sqrt{2}}(1, -i)$.

On the other hyperplane, $x^- = 0$ we get similar forms for the 'minus-movers:'

$$A_{iMb}(x^+, x_\perp) = \int \frac{d^2k_\perp}{(2\pi)^3}\frac{dk^-}{2k^-}\sum_\mu \left\{\epsilon_i^\mu \hat{a}_b(k^-, k_\perp, \mu)e^{-i\hat{k}.x} + \epsilon_i^{*\mu} \hat{a}_b^\dagger(k^-, k_\perp, \mu)e^{+i\hat{k}.x}\right\} \tag{3.3}$$

$$\psi_-(x^+, x_\perp) = \int \frac{d^2k_\perp}{(2\pi)^3}\frac{dk^-}{2k^-}\sum_\mu \left\{\hat{b}(k^-, k_\perp)u_-(k^-, k_\perp, \mu)e^{-i\hat{k}.x}\right.$$

$$\left. + \hat{d}^\dagger(k^-, k_\perp)v_-(k^-, k_\perp, \mu)e^{+i\hat{k}.x}\right\} \tag{3.4}$$

Here , $i\hat{k}.x = \frac{i}{2}k^-x^+ - ik_\perp.x_\perp$ and the polarization vector is like before.

Since the fields A^+, A^- go into defining the operators $\mathcal{H}$ and $\mathcal{K}$, we need to know their behaviour on $x^+ = 0$ and $x^- = 0$. Furthermore, to identify uniquely $\mathcal{H}$ and $\mathcal{K}$, we need $\mathcal{M}$ to be zero on the initializing surfaces.

On the surface $x^+ = 0$, our solution to $\mathcal{M} = 0$ is to set $C_\pi = A_P^+ = 0$. This is (with some rearrangement)

$$\partial^+(-\frac{1}{2}\partial^+ A^-_{Pa} - \partial_i A^i_{Pa}) = +g f_{abc} A^b_{Pi}\partial^+ A^{ic}_P - g\frac{v^+}{v^-}\psi^\dagger_+ T_a \psi_+ \tag{3.5}$$

Since on this surface the fields depend on $(x^-, x_\perp)$, we get a great simplification. As a matter of fact , we can easily solve this equation and obtain

$$A^-_{Pa} = -\frac{2}{(i\partial^+)} i\partial_i A^i_{Pa} - \frac{2}{(i\partial^+)^2} i g f_{abc} A^b_{Pi}(i\partial^+)A^{ic}_P - \frac{2}{(i\partial^+)^2} g\frac{v^+}{v^-}\psi^\dagger_+ T_a\psi_+ \tag{3.6}$$

To fully define these fields, we need to define the operators $((i\partial^+)^{-1}$ and $((i\partial^+)^2)^{-1}$. We use the definitions of the previous papers [3], [4], [5] and [6] .

For the fields on $x^- = 0$, we get similar results, noting though that the fields depend only on $(x^+, x_\perp)$. We get $\mathcal{M} = 0$ by setting $A^-_M = 0 = C_\rho$ and obtain

$$A^+_{Ma} = -\frac{2}{(i\partial^-)} i\partial_i A^i_{Ma} - \frac{2}{(i\partial^-)^2} i g f_{abc} A^b_{Mi}(i\partial^-)A^{ic}_M - \frac{2}{(i\partial^-)^2} g\frac{v^-}{v^+}\psi^\dagger_- T_a\psi_- \tag{3.7}$$

Again, to fully define this operator, we need to specify what do we mean by $(i\partial^-)^{-1}$ and $(i\partial^-)^{-2}$. We also follow the approach of the previous work [3], [4], [5] and [6] . The main difference with [1] is the presence of $\frac{v^+}{v^-}$ $(\frac{v^-}{v^+})$ in the definitions for A^-_{Pa} (A^+_{Ma}).

4. Perturbative Solution for Heavy Quark Effective Theory

We want to develop now a perturbation theory. Unlike the previous work [4], here we expand in $\frac{1}{m_Q}$ as well as in the coupling g . Using the results of the previous sections, we obtain the following forms for the Hamiltonians

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I + \frac{1}{2m_Q}\mathcal{H}_1 \tag{4.1}$$

where

$$\mathcal{H}_0 = \frac{1}{2}(\partial^i A^j_{Pa})^2 - \psi^\dagger_+ (\frac{i\partial^+}{2} - \frac{iv_i\partial^i}{v^-})\psi_+ \tag{4.2}$$

$$H_I = \mathcal{H}_{AAA} + \mathcal{H}_{q\bar{q}A} + \mathcal{H}_{AAAA} + \mathcal{H}_{q\bar{q}AA} + \mathcal{H}_{q\bar{q}q\bar{q}} \tag{4.3}$$

with

$$\mathcal{H}_{AAA} = igf^{abc}A^i_{Pa}A^j_{Pb}\partial^i A^j_{Pc} - igf^{abc}i\partial_i A^i_{Pa}\frac{1}{i\partial^+ i}A^j_{Pb}i\partial^+ A^j_{Pc} \tag{4.4}$$

$$\mathcal{H}_{q\bar{q}A} = g\frac{v^+}{v^-}\psi^\dagger_+ T^a \frac{1}{i\partial^+} i\partial_i A^i_{Pa}\psi_+ - \frac{g}{2v^-}\psi^\dagger_+\sigma_i A^i_{Pa}T^a\sigma_j i\partial_j\psi_+ - \frac{g}{2v^-}\psi^\dagger_+\sigma_j i\partial_j\sigma_i A^i_{Pa}T^a\psi_+$$

$$+\frac{gv^+}{v^-}\psi^\dagger_+ i\partial_k A^k_{Pa}T^a\frac{1}{i\partial^+}\psi_+ - \frac{g}{2v^-}\psi^\dagger_+\sigma_i\sigma_j i\partial_j A^i_{Pa}T^a\psi_+ - \frac{g}{2v^-}\psi^\dagger_+\sigma_i A^i_{Pa}T^a\sigma_j i\partial_j\psi_+$$

$$+2g\psi^\dagger_+ T^a\psi_+\frac{1}{i\partial^+}i\partial_i A^i_{Pa} \tag{4.5}$$

$$\mathcal{H}_{AAAA} = \frac{g^2}{4}f^{abc}f^{ade}A^i_{Pb}A^j_{Pc}A^i_{Pd}A^j_{Pe} + \frac{g^2}{2}f^{abc}f^{ade}\frac{1}{(i\partial^+)}A^i_{Pb}(i\partial^+)A^i_{Pc}\frac{1}{(i\partial^+)}A^j_{Pd}(i\partial^+)A^j_{Pe} \tag{4.6}$$

$$\mathcal{H}_{q\bar{q}AA} = +g^2\frac{v^+}{v^-}\psi^\dagger_+ T^a\psi_+\frac{1}{(i\partial^+)^2}if_{abc}A^b_{Pi}(i\partial^+)A^{ci}_P - g^2\frac{v^+}{v^-}if_{abc}A^b_{iP}A^{ic}_P\frac{1}{i\partial^+}\psi^\dagger_+ T^a\psi_+$$

$$+g^2\psi_+^\dagger\sigma_i A^i_{Pa}T^a\frac{1}{i\partial^+}\sigma_j A^j_{Pb}T^b\psi_+ \tag{4.7}$$

$$\mathcal{H}_{q\bar{q}q\bar{q}} = g^2\frac{v^+}{v^-}\psi_+^\dagger T^a\psi_+\frac{1}{(i\partial^+)^2}\psi_+^\dagger T^a\psi_+ - \frac{g^2}{2}(\frac{v^+}{v^-})^2\frac{1}{i\partial^+}\psi_+^\dagger T^a\psi_+\frac{1}{i\partial^+}\psi_+^\dagger T^a\psi_+ \tag{4.8}$$

and

$$\mathcal{H}_1 = -\frac{1}{v^-}\psi_+^\dagger(iD_\perp)^2\psi_+ - \frac{g}{4v^-}\psi_+^\dagger i\gamma_+\gamma_i F^{+i}\psi_+ \tag{4.9}$$

In a parallel notation to $\mathcal{H}_I$, we can write this as

$$\mathcal{H}_1 = \mathcal{H}_{1q\bar{q}} + \mathcal{H}_{1q\bar{q}A} + \mathcal{H}_{1q\bar{q}AA} + \mathcal{H}_{1q\bar{q}AAA} + \mathcal{H}_{1q\bar{q}AAAA}$$

$$+\mathcal{H}_{1q\bar{q}q\bar{q}} + \mathcal{H}_{1q\bar{q}q\bar{q}A} + \mathcal{H}_{1q\bar{q}q\bar{q}AA} + \mathcal{H}_{1q\bar{q}q\bar{q}AAA} + \mathcal{H}_{1q\bar{q}q\bar{q}AAAA} + \mathcal{H}_{1q\bar{q}q\bar{q}q\bar{q}} \tag{4.10}$$

We got

$$\mathcal{H}_{1q\bar{q}} = -\frac{1}{v^-}\psi_+^\dagger\{(i\partial^+)(i\partial^-)-(i\partial_i)^2-(v^+)^2(i\partial^-)^2-(v^-)^2(i\partial^+)^2-v_iv_j(i\partial^i)(i\partial^j)-2(v^+v^-)(i\partial^+i\partial^-)$$

$$+2(v^+v^i)(i\partial^-)(i\partial_i) + 2(v^+v^i)(i\partial^+)(i\partial_i)\}\psi_+ \tag{4.11}$$

$$\mathcal{H}_{1q\bar{q}A} = -\frac{1}{v^-}\psi_+^\dagger\{-gT^a i\partial_i A^i_{Pa} - gT^a A^i_{Pa}i\partial_i - v_iv_j gT^a i\partial^i A^j_{Pa} - v_iv_j gT^a A^i_{Pa}i\partial^j$$

$$+v^+v_j gT^a i\partial^- A^j_{Pa} + v^+v_i gT^a A^i_{Pa}i\partial^- + v^-v_j gT^a i\partial^+ A^j_{Pa} + v^-v_i gT^a A^i_{Pa}i\partial^+$$

$$+g(1-2v^+v^-)T^a i\partial_i A^i_{Pa}+g(1-2v^+v^-)T^a\frac{1}{i\partial^+}i\partial_i A^i_{Pa}(i\partial^+)-(v^+)^2 gT^a(i\partial^-)\frac{1}{i\partial^+}i\partial_i A^i_{Pa}$$

$$-(v^+)^2 gT^a\frac{1}{i\partial^+}i\partial_i A^i_{Pa}i\partial^- - v^+v^i gT^a i\partial_i\frac{1}{i\partial^+}i\partial_i A^i_{Pa} - v^+v^i gT^a\frac{1}{i\partial^+}i\partial_i A^i_{Pa}i\partial_i + \frac{g}{4}T^a\gamma_+\gamma_i i\partial^+ A^i_{Pa}\}\psi_+ \tag{4.12}$$

$$\mathcal{H}_{1q\bar{q}AA}=-\frac{1}{v^-}\psi_+^\dagger\{-g^2T^aT^bA_{iPa}A_{iPb}-v^iv^jgT^aT^bA_{iPa}A_{jPb}+g^2(1-2v^+v^-)if_{abc}T^a\frac{1}{(i\partial^+)}A_{iP}^bi\partial^+A_P^{ic}$$

$$+g^2(1-2v^+v^-)if_{abc}T^a\frac{1}{(i\partial^+)^2}A_{iP}^b(i\partial^+)A_P^{ic}(i\partial^+)+g^2v^+v^iif_{abc}T^a(i\partial^-)\frac{1}{(i\partial^+)^2}A_{iP}^b(i\partial^+)A_P^{ic}$$

$$+g^2v^+v^iif_{abc}T^a\frac{1}{(i\partial^+)^2}A_{iP}^bi\partial^+A_P^{ic}i\partial^-+2g^2v^+v^iT^aT^b\frac{1}{i\partial^+}i\partial_jA_{Pa}^jA_{iPb}+2g^2v^+v^iT^aT^bA_{iPa}\frac{1}{i\partial^+}i\partial_jA_{Pb}^j$$

$$-4(v^+)^2T^aT^b\frac{1}{i\partial^+}i\partial_iA_{Pa}^i\frac{1}{i\partial^+}i\partial_jA_{Pb}^j\}\psi_+ \tag{4.13}$$

$$\mathcal{H}_{1q\bar{q}AAA}=-\frac{1}{v^-}\psi_+^\dagger\{+2v^+v^ig^2if_{ade}T^aT^b\frac{1}{(i\partial^+)^2}A_{iP}^d(i\partial^+)A_P^{je}A_{jPb}$$

$$+2v^+v^ig^2if_{bde}T^aT^bA_{jPa}\frac{1}{(i\partial^+)^2}A_{jP}^d(i\partial^+)A_P^{ie}-4g(v^+)^2T^aT^bif_{bde}\frac{1}{i\partial^+}i\partial_iA_{Pa}^i\frac{1}{(i\partial^+)^2}A_{jP}^d(i\partial^+)A_P^{je}$$

$$-4g(v^+)^2T^aT^bif_{ade}\frac{1}{(i\partial^+)^2}A_{jP}^d(i\partial^+)A_P^{je}\frac{1}{i\partial^+}i\partial_iA_{Pb}^i\}\psi_+ \tag{4.14}$$

$$\mathcal{H}_{1q\bar{q}AAAA}=-\frac{1}{v^-}\psi_+^\dagger\{-4(v^+)^2g^2T^aT^bif_{acd}if_{bef}\frac{1}{(i\partial^+)^2}A_{iP}^c(i\partial^+)A_P^{id}\frac{1}{(i\partial^+)^2}A^ejP(i\partial^+)A_P^{jf}\}\psi_+ \tag{4.15}$$

$$\mathcal{H}_{1q\bar{q}q\bar{q}}=-\frac{1}{v^-}\psi_+^\dagger\{-g^2\frac{v^+}{v^-}(1-2v^+v^-)\frac{1}{(i\partial^+)}\psi_+^\dagger T_a\psi_+-g^2\frac{v^+}{v^-}(1-2v^+v^-)$$

$$\frac{1}{(i\partial^+)^2}\psi_+^\dagger T_a\psi_+(i\partial^+)+g^2\frac{v^+}{v^-}(v^+)^2(i\partial^-)\frac{1}{(i\partial^+)^2}\psi_+^\dagger T_a\psi_++g^2\frac{v^+}{v^-}(v^+)^2$$

$$\frac{1}{(i\partial^+)^2}\psi_+^\dagger T_a\psi_+(i\partial^-)+g^2\frac{v^+}{v^-}(v^+v^i)(i\partial_i)\frac{1}{(i\partial^+)^2}\psi_+^\dagger T_a\psi_++g^2\frac{v^+}{v^-}(v^+v^i)\frac{1}{(i\partial^+)^2}\psi_+^\dagger T_a\psi_+(i\partial_i)\}\psi_+ \tag{4.16}$$

$$\mathcal{H}_{1q\bar{q}q\bar{q}A}=-\frac{1}{v^-}\psi_+^\dagger\{-2g^3\frac{v^+}{v^-}(v^+v^i)T^aT^b\frac{1}{(i\partial^+)^2}\psi_+^\dagger T_a\psi_+A_{iPb}$$

$$-2g^3\frac{v^+}{v^-}(v^+v^i)T^aT^bA_{iPa}\frac{1}{(i\partial^+)^2}\psi_+^\dagger T_b\psi_++4g\frac{v^+}{v^-}(v^+)^2T^aT^b\frac{1}{(i\partial^+)}i\partial_iA_{Pa}^i\frac{1}{(i\partial^+)^2}\psi_+^\dagger T_b\psi_+$$

$$+4g\frac{v^+}{v^-}(v^+)^2T^aT^b\frac{1}{(i\partial^+)^2}\psi_+^\dagger T_a\psi_+\frac{1}{(i\partial^+)}i\partial_jA^j_{Pb}\}\psi_+ \tag{4.17}$$

$$\mathcal{H}_{1q\bar{q}q\bar{q}AA} = -\frac{1}{v^-}\psi_+^\dagger\{-4g\frac{v^+}{v^-}(v^+)^2T^aT^bif_{ade}\frac{1}{(i\partial^+)^2}A^d_{jP}(i\partial^+)A^{je}_P\frac{1}{i\partial^+}i\partial_iA^i_{Pb}\frac{1}{(i\partial^+)^2}\psi_+^\dagger T_a\psi_+$$

$$-4g\frac{v^+}{v^-}(v^+)^2T^aT^bif_{bde}\frac{1}{(i\partial^+)^2}\psi_+^\dagger T_a\psi_+\frac{1}{(i\partial^+)^2}A^d_{jP}(i\partial^+)A^{je}_P\frac{1}{i\partial^+}i\partial_iA^i_{Pa}\}\psi_+ \tag{4.18}$$

$$\mathcal{H}_{1q\bar{q}q\bar{q}q\bar{q}} = -\frac{1}{v^-}\psi_+^\dagger\{-4g^2\frac{{v^+}^2}{v^-}(v^+)^2T^aT^b\frac{1}{(i\partial^+)^2}\psi_+^\dagger T_a\psi_+\frac{1}{(i\partial^+)^2}\psi_+^\dagger T_b\psi_+\}\psi_+ \tag{4.19}$$

Similarly, for $\mathcal{K}$ we get the same results, with $+\rightarrow-$ and $P\rightarrow M$ in the above expressions.

In the case of heavy quark effective theory, we have two parameters in which we can do a perturbative expansion: $\alpha(m_Q)$ and $\frac{1}{2m_Q}$. We organize our perturbation theory as follows: we first expand in zeroth order in $\frac{1}{2m_Q}$ but to first order in $\alpha(m_Q)$; then, we compute the terms of order $\frac{1}{2m_Q}$ and again to first order in $\alpha(m_Q)$, following [10].

For this, we use the perturbation theory in Penrose quantization which we developed already [4]. Specifically we can define then the wave function renormalization as follows :

$$|\Psi>=UV|\phi>=\{|\phi>+\sum_{n_1}\frac{|n_1><n_1|P_I^-|n_1>}{P_0^--P_{n_1}^-+i\epsilon}+\sum_{m_1}\frac{|m_1><m_1|P_I^+|m_1>}{P_0^+-P_{m_1}^++i\epsilon}$$

$$+\sum_{n_1,n_2}\frac{|n_1><n_1|P_I^-|n_2><n_2|P_I^-|n_2>}{(P_0^--P_{n_1}^-+i\epsilon)(P_0^--P_{n_2}^-+i\epsilon)}+\sum_{m_1,m_2}\frac{|m_1><m_1|P_I^+|m_2><m_2|P_I^+|m_2>}{(P_0^+-P_{n_1}^++i\epsilon)(P_0^+-P_{n_2}^++i\epsilon)}$$

$$+2\sum_{n_1,m_1}\frac{|n_1><n_1|P_I^-|m_1><m_1|P_I^+|m_1>}{(P_0^--P_{n_1}^-+i\epsilon)(P_0^+-P_{m_1}^++i\epsilon)}...\} \tag{4.20}$$

where U, V are unitary transformations which undress the operators and go to the interaction representation [14] (Here, V is just the x^- counterpart of U). P_0^- is the free part of P^- and P_I^- is the interacting part (the $\alpha(m_Q)$ dependent but m_Q independent part), while $P_{n_1}^-$ is the P_0^- of the intermediate state n_1 .

In the case where the initial state is a single particle state, we can extract the wave function renormalization factor Z_ϕ:

$$|\Psi>= UV|\phi>= Z_\phi^{1/2}\{|\phi> + \sum_{n_1}' \frac{|n_1><n_1|P_I^-|n_1>}{P_0^- - P_{n_1}^- + i\epsilon} + \sum_{m_1}' \frac{|m_1><m_1|P_I^+|m_1>}{P_0^+ - P_{m_1}^+ + i\epsilon} \tag{4.21}$$

where the notation $\sum'$ indicates summing over all states **except** the initial state ϕ. Using the normalization condition,

$$<\phi|V^{-1}U^{-1}UV|\phi'>= \delta_{\phi,\phi'} = \delta_{\Psi,\Psi'} \tag{4.22}$$

we get

$$Z_\phi^{-1} = 1 + \sum_{n_1}' \frac{|<n_1|P_I^-|\phi>|^2}{(P_0^- - P_{n_1}^- + i\epsilon)^2} + \sum_{m_1}' \frac{|<m_1|P_I^+|\phi>|^2}{(P_0^+ - P_{n_1}^+ + i\epsilon)^2}$$

$$+2\sum_{n_1} \frac{<\phi|P_I^-|n_1><n_1|P_I^+|\phi>}{(P_0^- - P_{n_1}^- + i\epsilon)(P_0^+ - P_{m_1}^+ + i\epsilon)} \ldots \tag{4.23}$$

The mass and coupling renormalization is obtained from the expression

$$<\Psi_F|O|\Psi_I>=<\phi_f|O|\phi_i>$$

$$+\sum_{n_1} \frac{<\phi_f|P_I^-|n_1><n_1|O|\phi_i>}{(P_0^- - P_{n_1}^- + i\epsilon)} + \sum_{m_1} \frac{<\phi_f|P_I^+|m_1><m_1|O|\phi_i>}{(P_0^+ - P_{m_1}^+ + i\epsilon)}$$

$$+\sum_{n_1,m_1} \frac{<\phi|P_I^-|n_1><n_1|P_I^+|m_1><m_1|O|\phi_i>}{(P_0^- - P_{n_1}^- + i\epsilon)(P_0^+ - P_{m_1}^+ + i\epsilon)} \tag{4.24}$$

where O is some operator .

We can define the mass shifts easily now, just by using [(4.24)] for the same initial , final state, and by using P^- and P^+ for the operator O. We get :

$$\delta M^2 = P_0^+ \delta P^- + P_0^- \delta P^+$$

where δP^-, δP^+ are defined thus:

$$\delta P^- =< \Psi | P_I^- | \Psi >$$

$$=< \phi | P_I^- | \phi > + \sum_{n_1} \frac{< \phi | P_I^- | n_1 >< n_1 | P_I^- | \phi >}{(P_0^- - P_{n_1}^- + i\epsilon)}$$

$$+ \sum_{n_1} \frac{< \phi | P_I^- | n_1 >< n_1 | P_I^+ | \phi >}{(P_0^+ - P_{n_1}^+ + i\epsilon)} \tag{4.25}$$

Likewise, for δP^+ we get

$$\delta P^+ =< \Psi | P_I^+ | \Psi >$$

$$=< \phi | P_I^+ | \phi > + \sum_{m_1} \frac{< \phi | P_I^+ | m_1 >< m_1 | P_I^+ | \phi >}{(P_0^+ - P_{m_1}^+ + i\epsilon)}$$

$$+ \sum_{m_1} \frac{< \phi | P_I^- | m_1 >< m_1 | P_I^+ | \phi >}{(P_0^+ - P_{m_1}^+ + i\epsilon)} \tag{4.26}$$

Then the mass renormalization becomes

$$\delta M^2 = P_0^+ < \phi | P_I^- | \phi > + P_0^+ \sum_{n_1} \frac{< \phi | P_I^- | n_1 >< n_1 | P_I^- | \phi >}{(P_0^- - P_{n_1}^- + i\epsilon)}$$

$$+ P_0^+ \sum_{n_1} \frac{< \phi | P_I^- | n_1 >< n_1 | P_I^+ | \phi >}{(P_0^+ - P_{n_1}^+ + i\epsilon)}$$

$$+P_0^- < \phi|P_I^+|\phi> + P_0^- \sum_{m_1} \frac{<\phi|P_I^+|m_1><m_1|P_I^+|\phi>}{(P_0^+ - P_{m_1}^+ + i\epsilon)}$$

$$+P_0^- \sum_{m_1} \frac{<\phi|P_I^-|m_1><m_1|P_I^+|\phi>}{(P_0^+ - P_{m_1}^+ + i\epsilon)} \tag{4.27}$$

We can also see now that the on-shell wave function renormalization is just like in [10]

$$Z_\phi^{-1} = 1 - (\frac{\partial \delta P^-}{\partial P_0^-})_{v.k=0} - (\frac{\partial \delta P^+}{\partial P_0^+})_{v.k=0} \tag{4.28}$$

where the heavy quark momentum p is parametrized as usual, $p^\mu = m_Q v^\mu + k^\mu$.

As for coupling constant renormalization, we just take the operator $(P_I^- + P_I^+)$ between different initial, final states. We get

$$<\Psi_F|(P_I^- + P_I^+)|\Psi_I> = Z_g (Z_I Z_F)^{1/2} \{ <\phi_f|P_I^-|\phi_i> + <\phi_f|P_I^+|\phi_i>$$

$$+ \sum_{n_1} \frac{<\phi_f|P_I^-|n_1><n_1|(P_I^- + P_I^+)|\phi_i>}{(P_0^- - P_{n_1}^- + i\epsilon)} + \sum_{m_1} \frac{<\phi_f|P_I^+|m_1><m_1|P_I^- + P_I^+|\phi_i>}{(P_0^+ - P_{m_1}^+ + i\epsilon)}$$

$$+ \sum_{n_1,n_2} \frac{<\phi_f|P_I^-|n_1><n_1|P_I^-|n_2><n_2|(P_I^- + P_I^+)|\phi_i>}{(P_0^- - P_{n_1}^- + i\epsilon)(P_0^- - P_{n_2}^- + i\epsilon)}$$

$$+ \sum_{m_1,m_2} \frac{<\phi_f|P_I^+|m_1><m_1|P_I^+|m_2><m_2|(P_I^- + P_I^+)|\phi_i>}{(P_0^+ - P_{m_1}^+ + i\epsilon)(P_0^+ - P_{m_2}^+ + i\epsilon)}$$

$$+ \sum_{n_1,m_2} \frac{<\phi_f|P_I^-|n_1><n_1|P_I^-|m_2><m_2|(P_I^- + P_I^+)|\phi_i>}{(P_0^- - P_{n_1}^- + i\epsilon)(P_0^+ - P_{m_2}^+ + i\epsilon)}$$

$$+ \sum_{m_1,n_2} \frac{<\phi_f|P_I^+|m_1><m_1|P_I^-|n_2><n_2|(P_I^- + P_I^+)|\phi_i>}{(P_0^+ - P_{m_1}^+ + i\epsilon)(P_0^- - P_{n_2}^- + i\epsilon)} \} \tag{4.29}$$

5. Heavy-Light Currents

Our first application will be to heavy-light current interaction

$$g V_\mu A^\mu = g \overline{q} \gamma_\mu Q A^\mu \tag{5.1}$$

Following the discussion of [10], we see that we need to compare the QCD perturbative calculation of this current interaction with the $HQET$ calculation to same order in $\alpha(m_Q)$ and $\frac{1}{2m_Q}$. There might be an extra difficulty here in that we don't work in the background field quantization method, hence we need to carry out the gauge coupling and gauge field renormalization as well (Recall that in [10] it was pointed out that in the background field method, the coupling renormalization is related to the gauge field renormalization , $Z_g = Z_A^{-\frac{1}{2}}$, so that the combination gA stays unaffected by renormalization!).

Instead, in our work we need to compute the following renormalization constants : in QCD, the set $Z_{QCD} = \{Z_g, Z_A, Z_q, Z_Q\}$; in $HQET$, the set $Z_{HQET} = \{Z_g, Z_A, Z_q, Z_h\}$. For the QCD interaction, we obtain

$$< g V_\mu A^\mu >_{QCD} = Z_g Z_A^{1/2} Z_Q^{1/2} Z_q^{1/2} g \overline{u}_q \gamma_\mu u_Q A^\mu \tag{5.2}$$

For $HQET$, we expand the current interaction in local operators and Wilson coefficients $C_i(\mu)$:

$$< g V_\mu A^\mu >_{HQET} = \sum_{i=1,2} C_i(\mu) g J_i A + O(\frac{1}{m_Q}) \tag{5.3}$$

Here, μ is a renormalization scale such that $\Lambda_{QCD} \ll \mu \ll m_Q$. This renormalized current J_i can be written as

$$J_i = Z_{ij}^{-1} Z_g Z_A^{1/2} Z_h^{1/2} Z_q^{1/2} \overline{q} \Gamma_j h_v \tag{5.4}$$

where

$$\Gamma_1 = \gamma_\mu, \Gamma_2 = v_\mu \tag{5.5}$$

and Z_{ij}^{-1} is a matrix of renormalization factors due to the mixing of operators coming from renormalization.

We compare then the QCD calculation with the $HQET$ calculation by using the following relationship

$$< gV_\mu A^\mu >_{QCD} = C_0(\mu) < gV_\mu^0 A^\mu >_{HQET} + \frac{C_1(\mu)}{2m_Q} < gV_\mu^1 A^\mu >_{HQET} + ... \tag{5.6}$$

To do this, we need to compute the two sets of renormalization constants, Z_{QCD} and Z_{HQET}. The first set was already calculated in [1]. The latter set will be computed in an upcoming paper [15] .

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