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Quantization of Gauge Field Theories on the Front-Form without Gauge Constraints II: Perturbation Theory and Renormalization*

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ABSTRACT

In this paper we work out the perturbation theory and the renormalization procedure for Abelian gauge fields quantized following the method inspired by the work of Penrose. We compute the fermion mass and wave-function renormalization as well as the photon mass and wave-function renormalization to one-loop order and find complete agreement with the usual canonical quantization results. We also prove the Ward identity, to one-loop order. We use transverse dimensional regularization for these results.

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1. Introduction

In previous papers [1] [2] [3] [4] [5] a quantization was introduced which treated x^+ and x^- on equal footing [6] . In the present paper we work out the perturbation theory and the renormalization method for observables up to first order in the gauge coupling α.

In honor of the first person who pointed out that such a construction as presented here and in [1] contains no constraints, we would like to call this Penrose quantization. [7]

The outline is as follows: we start where the previous paper left off, and show how to construct the generators P^- and P^+ for evolution along $x^+ = 0$ and $x^- = 0$ respectively. We do this using perturbation theory, being unable to construct a non-perturbative solution. We then show how the renormalization procedure works out, following closely the method of Harindranth and Zhang. [8] . We do this for several physical observables.

2. Reduced Phase Space Quantization of QED

The Lagrangean of QED is :

$$\mathcal{L} = -\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\partial^\nu\overline{\psi}\gamma_\nu\psi - \overline{\psi}\frac{i}{2}\partial^\nu\gamma_\nu\psi - m\overline{\psi}\psi + \mathcal{L}_I \tag{2.1}$$

where $\mathcal{L}_I$ is the interaction part of the Lagrangean:

$$\mathcal{L}_I = -e\overline{\psi}\gamma^\nu\psi A_\nu \tag{2.2}$$

Previous work [1] showed how to construct the Hamiltonian densities $\mathcal{H}$ and $\mathcal{K}$ (after integrating by parts):

$$\mathcal{H} = \{\frac{1}{2}(B^{-2}+(E^{-2}))+2e\psi_+^\dagger A_+\psi_+ +2e\psi_-^\dagger A_-\psi_- +\psi_+^\dagger\gamma_0\gamma_i\frac{i\partial_i}{2}\psi_- - e\psi_+^\dagger\gamma_0\gamma_i\frac{A^i}{2}\psi_-$$

$$+\psi_-^\dagger\gamma_0\gamma_i\frac{i\partial_i}{2}\psi_+ - e\psi_-^\dagger\gamma_0\gamma_i\frac{A^i}{2}\psi_+ + m\psi_+^\dagger\frac{\gamma^-}{4}\psi_- + m\psi_-^\dagger\frac{\gamma^+}{4}\psi_+\} \qquad (2.3)$$

$$\mathcal{K} = \{\frac{1}{2}(B^{+2}+(E^{+2}))+2e\psi_-^\dagger A_-\psi_- +2e\psi_+^\dagger A_+\psi_+ +\psi_-^\dagger\gamma_0\gamma_i\frac{i\partial_i}{2}\psi_+ - e\psi_-^\dagger\gamma_0\gamma_i\frac{A^i}{2}\psi_+$$

$$+\psi_+^\dagger\gamma_0\gamma_i\frac{i\partial_i}{2}\psi_- - e\psi_+^\dagger\gamma_0\gamma_i\frac{A^i}{2}\psi_- + m\psi_-^\dagger\frac{\gamma^+}{4}\psi_+ + m\psi_+^\dagger\frac{\gamma^-}{4}\psi_-\} \qquad (2.4)$$

where $B^- = B^+ = F^{12}$ and $E^- = -E^+ = -\frac{1}{2}F^{+-}$.

Going now to the quantum commutators, we get the following relations for fields at equal $x^+ = y^+$, the usual front-form 'time' :

$$[A_P^i(x^+,x^-,x_\perp), A_P^j(y^+,y^-,y_\perp)]_{x^+=y^+} = \frac{-i}{4}\epsilon(x^- - y^-)\delta^2(x_\perp - y_\perp)\delta^{ij} \qquad (2.5)$$

$$\left\{\psi_+(x^+,x^-,x_\perp), \psi_+(y^+,y^-,y_\perp)\right\}_{x^+=y^+} = \Lambda_+\delta(x^- - y^-)\delta^2(x_\perp - y_\perp) \qquad (2.6)$$

So the physical (quantized) degrees of freedom on $x^+ = 0$ are A_P^i , ψ_+ and $\psi_+^\dagger$.

For fields at equal $x^- = y^-$, a new front-form 'time', we get:

$$[A_M^i(x^+,x^-,x_\perp), A_M^j(y^+,y^-,y_\perp)]_{x^-=y^-} = \frac{-i}{4}\epsilon(x^- - y^-)\delta^2(x_\perp - y_\perp)\delta^{ij} \qquad (2.7)$$

$$\left\{\psi_-(x^+,x^-,x_\perp), \psi_-(y^+,y^-,y_\perp)\right\}_{x^-=y^-} = \Lambda_-\delta(x^+ - y^+)\delta^2(x_\perp - y_\perp) \qquad (2.8)$$

Here, the physical (quantized) degrees of freedom on $x^- = 0$ are A_M^i , ψ_- and $\psi_-^\dagger$. Note that A^+ and A^- do not enter in the list of physical (quantized) degrees of freedom.

Rather, they are determined from requiring that $\mathcal{M}$ is zero on the initializing surfaces. The solutions were found to be [1] :

$$A_P^+ = 0 \tag{2.9}$$

$$A_P^- = -\frac{2i\partial_i}{(i\partial^+)} A_P^i + \frac{4}{(i\partial^+)^2} e\psi_+^\dagger \psi_+ \tag{2.10}$$

on the surface $x^+ = 0$ and

$$A_M^- = 0 \tag{2.11}$$

$$A_M^+ = -\frac{2i\partial_i}{(i\partial^-)} A_M^i + \frac{4}{(i\partial^-)^2} e\psi_-^\dagger \psi_- \tag{2.12}$$

The equations of motion for the fermionic degrees are

$$i\partial^- \psi_+ = \gamma_0\gamma_i i\partial_i \psi_- - \gamma_0\gamma_i A_M^i e\psi_- + m\gamma_0\psi_- \tag{2.13}$$

as well as

$$i\partial^+ \psi_- = \gamma_0\gamma_i i\partial_i \psi_+ - \gamma_0\gamma_i A_P^i e\psi_+ + m\gamma_0\psi_+ \tag{2.14}$$

These can be solved easily. Thus, if we want to express ψ_- in terms of ψ_+, we have

$$\psi_- = \frac{1}{(i\partial^+)} \{\gamma_0\gamma_i i\partial_i - \gamma_0\gamma_i A^i e + m\gamma_0\} \psi_+ \tag{2.15}$$

Does this mean that we violate Weinberg's conservation of difficulty 'theorem ?' [9] It is not clear yet.

As in the previous papers [3], [1] the fields are defined on the initializing surfaces $x^+ = 0$, $x^- = 0$, and $x^+ = x^- = 0$ as follows:

$$A_i(x^+, x^-, x_\perp) = A_{iP}(x^-, x_\perp) + A_{iM}(x^+, x_\perp) - B_i(x_\perp) \tag{2.16}$$

and the fermion field is defined to be

$$\psi(x^+, x^-, x_\perp) = \psi_-(x^-, x_\perp) + \psi_+(x^+, x_\perp) \tag{2.17}$$

where the field B_i is just the gauge field along the hypersurface $x^+ = x^- = 0$:

$$A_{iP}(x^- = 0, x_\perp) = A_{iM}(x^+ = 0, x_\perp) = B_i(x_\perp) \tag{2.18}$$

We've added B_i in so that along that hypersurface we don't double count the field. Notice also that now along $x^+ = 0$, we have

$$A_i(x^+ = 0, x^-, x_\perp) = A_{iP}(x^-, x_\perp) \tag{2.19}$$

and along $x^- = 0$ we like-wise have

$$A_i(x^+, x^- = 0, x_\perp) = A_{iM}(x^+, x_\perp) \tag{2.20}$$

namely, that the total field A_i reduces to A_{iP} or A_{iM} along these surfaces, as it should, so this is a consistent definition.

Let us study now the Hamiltonian P^-. It is constructed out of the Hamiltonian density $\mathcal{H}$. But first, we must express all the fields in terms of the fields defined on $x^+ = 0$. We have

$$\mathcal{H} = \{\frac{1}{2}(B_P^{-2} + E_P^{-2}) + \psi_+^\dagger \psi_+ e A_P^- + \psi_-^\dagger \psi_- e A_P^+ + \psi_+^\dagger \gamma_0 \gamma_i \frac{i\partial_i}{2}\psi_- - e\psi_+^\dagger \gamma_0 \gamma_i \frac{A_P^i}{2}\psi_-$$

$$+\psi_-^\dagger \gamma_0 \gamma_i \frac{i\partial_i}{2}\psi_+ - e\psi_-^\dagger \gamma_0 \gamma_i \frac{A_P^i}{2}\psi_+ + m\psi_+^\dagger \frac{\gamma^-}{4}\psi_- + m\psi_-^\dagger \frac{\gamma^+}{4}\psi_+\} \tag{2.21}$$

where

$$B_P^- = (\partial^1 A_P^2 - \partial^2 A_P^1) \tag{2.22}$$

and

$$E_P^- = -\frac{1}{2}\partial^+ A_P^- \tag{2.23}$$

We follow the notation of [8] and observe that the fermionic fields are

$$\psi_+^\dagger = (\xi^\dagger, 0) \tag{2.24}$$

and

$$\psi_-^\dagger = (0, \eta^\dagger) \tag{2.25}$$

so that we have

$$\eta = \frac{1}{i\partial^+}\{i\partial_i \sigma_i - \sigma_i e A_P^i + im\}\xi \tag{2.26}$$

where σ_i are the 2x2 Pauli sigma matrices and now both spinors ξ and η are two-component spinors [8].

Substituting this in the equation [(2.21)] we get

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I \tag{2.27}$$

where

$$\mathcal{H}_0 = \frac{1}{2}A^i_P(\partial_i^2)A^i_P + \frac{1}{2}\xi^\dagger\frac{(-\partial_i^2+m^2)}{i\partial^+}\xi \tag{2.28}$$

while the interaction Hamiltonian density is much more complicated. (Notice the surprising factors of $\frac{1}{2}$ in $\mathcal{H}_0$. This turns out to be correct! See Appendix A for a detailed explanation).

we can write the interaction terms thus:

$$\mathcal{H}_I = \mathcal{H}_{e^+e^-A} + \mathcal{H}_{e^+e^-AA} + \mathcal{H}_{e^+e^-e^+e^-} \tag{2.29}$$

These pieces are as follows:

$$\mathcal{H}_{e^+e^-A} = -e\xi^\dagger\frac{i\partial_k}{i\partial^+}A^k_P\xi + \frac{e}{2}\xi^\dagger\sigma_i A^i_P\frac{1}{i\partial^+}(\sigma_j i\partial_j + im)\xi + \frac{e}{2}\xi^\dagger\frac{1}{i\partial^+}(\sigma_j i\partial_j - im)\sigma_i A^i_P\xi$$

$$-e\xi^\dagger i\partial_k A^k_P\frac{1}{i\partial^+}\xi + \frac{e}{2}\xi^\dagger\sigma_i(\sigma_j i\partial_j + im)\frac{1}{i\partial^+}A^i_P\xi + \frac{e}{2}\xi^\dagger\sigma_i A^i_P(\sigma_j i\partial_j - im)\frac{1}{i\partial^+}\xi$$

$$-2e\xi^\dagger\xi\frac{1}{i\partial^+}i\partial_i A^i_P \tag{2.30}$$

$$\mathcal{H}_{e^+e^-AA} = e^2\xi^\dagger\sigma_i A^i_P\frac{1}{i\partial^+}\sigma_j A^j_P\xi \tag{2.31}$$

$$\mathcal{H}_{e^+e^-e^+e^-} = 4e^2\xi^\dagger\xi\frac{1}{(i\partial^+)^2}\xi^\dagger\xi - 2e^2\frac{1}{(i\partial^+)}\xi^\dagger\xi\frac{1}{(i\partial^+)}\xi^\dagger\xi \tag{2.32}$$

We see that we have recovered Harindranath and Zhang's results, but with a difference [8]: where they had just the term corresponding to their $\Gamma^i_{q0}(p_1,p_2,k)$, we have $\frac{1}{2}$ of this **plus** $\frac{1}{2}$ of its complex comjugate. Note also that our result is real, compared to Harindranath and Zhang's result, which is complex, since they lack the complex conjugate piece. This will be seen to have profound consequences for perturbation theory.

Then the generators of x^+ translations are defined to be

$$P_0^- = \frac{1}{2}\int dx^- d^2x_\perp \mathcal{H}_0 \tag{2.33}$$

$$P_I^- = \frac{1}{2}\int dx^- d^2x_\perp \mathcal{H}_I \tag{2.34}$$

We obtain the similar forms for P^+ by just interchanging + with – and $_P$ with $_M$, as well as interchanging the spinors ξ with η. For instance, $P^+_{e^-e^+e^+e^-}$ is

$$\mathcal{P}^+_{e^+e^-e^+e^-} = \frac{1}{2}\int dx^+ d^2x_\perp \{4e^2\eta^\dagger\eta\frac{1}{(i\partial^-)^2}\eta^\dagger\eta - 2e^2\frac{1}{(i\partial^-)}\eta^\dagger\eta\frac{1}{(i\partial^-)}\eta\} \tag{2.35}$$

3. x^+- and x^--Ordered Expansions

[10] We follow now the approach of Bjorken, Kogut and Soper [10] as well as the approach of Harindranath and Zhang [8] to obtain a perturbative theory of QED , ordered in x^+ as well as in x^-, since here both of these coordinates are 'times.'

We have the operator U, which undresses the Heisenberg operators when we go to the interaction picture:

$$U(x^+, x_0^+) = T_+ exp\{\frac{-i}{2}\int_{x_0^+}^{x^+} d\tau H_I(\tau)\} \tag{3.1}$$

where T_+ is a x^+-ordering operator. We have a similar operator, called V, which does the x^- undressing:

$$V(x^-,x_0^-) = T_- exp\{\frac{-i}{2}\int_{x_0^-}^{x^-} d\sigma K_I(\sigma)\} \tag{3.2}$$

and here T_- is the x^--ordering operator. The full interacting Heisenberg operators O and state vectors $|\Psi>$ are defined thus

$$O(x^+,x^-) = U(0,x^+)V(0,x^-)O_I(x^+,x^-)V(x^-,0)U(x^+,0) \tag{3.3}$$

$$|\Psi>= U(0,x^+)V(0,x^-)|\Psi>_I \tag{3.4}$$

The equations of motions are

$$i\partial^-|\Psi>_I = H_I|\Psi>, \partial^- O = \frac{1}{i}[O_I, H_0] \tag{3.5}$$

We have similar results for the x^- evolution. The perturbative expansion is like the familiar expansion for quantum mechanics, namely we invoke the adiabatic expansion, and write, as in [8]

$$U_\epsilon(x^+,x_0^+) = \sum_{n=0}^{\infty}\frac{1}{n!}\int_{x_0^+}^{x^+} d\tau_1 ... \int_{x_0^+}^{x^+} d\tau_n e^{-\epsilon(|\tau_1|+...+|\tau_n|)} T_+[H_I(\tau_1)...H_I(\tau_n)] \tag{3.6}$$

Here, ϵ is an infinitesimal number which we take to zero at the end of the calculation. There is a similar expansion for $V_\epsilon(x^-,x_0^-)$.

Then, the perturbative expansion is as follows : the fully interacting state vector $|\Psi>$ is constructed out of $|\phi>$, the eigenvector of the free Hamiltonians

H_0 and K_0 (from now on we drop the subscript I on the states):

$$|\Psi >= UV|\phi > \tag{3.7}$$

where both U and V have expansions as in eq. [(3.6)]. Likewise, matrix elements of any operator O between initial state $|\Psi_i >$ and final state $|\Psi_f >$ is defined thus

$$< |\Psi_f|O(0)|\Psi_i >=< \phi_f|V^{-1}U^{-1}O(0)UV|\phi_i > \tag{3.8}$$

[11] [12] [13] [14]

4. Renormalization in the Penrose Quantization

We develop now a renormalization procedure based on perturbation theory which we have discussed above as well as some ideas from olde fashion perturbation theory. Note that since both P^- and P^+ are dynamical, this is quite a bit different than the usual front-form renormalization [8] ,[11],[12],[13],[14].

Following the arguments of olde fashioned perturbation theory, we can define then the wave-function renormalization as above (see also [8]):

$$|\Psi >= UV|\phi >= \{|\phi > + \sum_{n_1} \frac{|n_1 >< n_1|P_I^-|n_1 >}{P_0^- - P_{n_1}^- + i\epsilon} + \sum_{m_1} \frac{|m_1 >< m_1|P_I^+|m_1 >}{P_0^+ - P_{m_1}^+ + i\epsilon}$$

$$+ \sum_{n_1,n_2} \frac{|n_1 >< n_1|P_I^-|n_2 >< n_2|P_I^-|n_2 >}{(P_0^- - P_{n_1}^- + i\epsilon)(P_0^- - P_{n_2}^- + i\epsilon)}$$

$$+ \sum_{m_1,m_2} \frac{|m_1 >< m_1|P_I^+|m_2 >< m_2|P_I^+|m_2 >}{(P_0^+ - P_{n_1}^+ + i\epsilon)(P_0^+ - P_{n_2}^+ + i\epsilon)}$$

$$+2 \sum_{n_1,m_1} \frac{|n_1 >< n_1|P_I^-|m_1 >< m_1|P_I^+|m_1 >}{(P_0^- - P_{n_1}^- + i\epsilon)(P_0^+ - P_{m_1}^+ + i\epsilon)} ...\} \tag{4.1}$$

where U, V are unitary transformations which undress the operators and go to the interaction representation [8] as were described above. P_0^- is the free part of P^- and P_I^- is the interacting part, while $P_{n_1}^-$ is the P_0^- of the intermediate state $|n_1>$. These states $|n_1>$, $|n_2>$ are properly symmetrized (antisymmetrized) states for identical bosons (fermions) , eigenstates of the free Hamiltonians.

In the case where the initial state is a single particle state, we can extract the wave-function renormalization factor Z_ϕ [10]:

$$|\Psi>= UV|\phi>= Z_\phi^{1/2}\{|\phi> + \sum_{n_1}{}^{'} \frac{|n_1><n_1|P_I^-|n_1>}{P_0^- - P_{n_1}^- + i\epsilon} + \sum_{m_1}{}^{'} \frac{|m_1><m_1|P_I^+|m_1>}{P_0^+ - P_{m_1}^+ + i\epsilon} \tag{4.2}$$

where the notation $\sum'$ indicates summing over all states **except** the initial state ϕ. Using the normalization condition,

$$<\phi|V^{-1}U^{-1}UV|\phi'>= \delta_{\phi,\phi'} = \delta_{\Psi,\Psi'} \tag{4.3}$$

we get

$$Z_\phi^{-1} = 1 + \sum_{n_1}{}^{'} \frac{|<n_1|P_I^-|\phi>|^2}{(P_0^- - P_{n_1}^- + i\epsilon)^2} + \sum_{m_1}{}^{'} \frac{|<m_1|P_I^+|\phi>|^2}{(P_0^+ - P_{n_1}^+ + i\epsilon)^2}$$

$$+2\sum_{n_1} \frac{<\phi|P_I^-|n_1><n_1|P_I^+|\phi>}{(P_0^- - P_{n_1}^- + i\epsilon)(P_0^+ - P_{m_1}^+ + i\epsilon)} ... \tag{4.4}$$

The mass and coupling renormalization is obtained from the expression

$$<\Psi_F|O|\Psi_I>=<\phi_f|O|\phi_i>$$

$$+\sum_{n_1} \frac{<\phi_f|P_I^-|n_1><n_1|O|\phi_i>}{(P_0^- - P_{n_1}^- + i\epsilon)} + \sum_{m_1} \frac{<\phi_f|P_I^+|m_1><m_1|O|\phi_i>}{(P_0^+ - P_{m_1}^+ + i\epsilon)}$$

$$+\sum_{n_1,m_1} \frac{<\phi|P_I^-|n_1><n_1|P_I^+|m_1><m_1|O|\phi_i>}{(P_0^- - P_{n_1}^- + i\epsilon)(P_0^+ - P_{m_1}^+ + i\epsilon)} \tag{4.5}$$

where O is some operator .

We can define the mass shifts easily now, just by using [(4.5)] for the same initial , final state, and by using P^- and P^+ for the operator O. We get :

$$\delta M^2 = P_0^+ \delta P^- + P_0^- \delta P^+$$

where δP^-, δP^+ are defined thus:

$$\delta P^- =< \Psi | P_I^- | \Psi >$$

$$=< \phi | P_I^- | \phi > + \sum_{n_1} \frac{< \phi | P_I^- | n_1 >< n_1 | P_I^- | \phi >}{(P_0^- - P_{n_1}^- + i\epsilon)}$$

$$+ \sum_{n_1} \frac{< \phi | P_I^- | n_1 >< n_1 | P_I^+ | \phi >}{(P_0^+ - P_{n_1}^+ + i\epsilon)} \tag{4.6}$$

Likewise, for δP^+ we get

$$\delta P^+ =< \Psi | P_I^+ | \Psi >$$

$$=< \phi | P_I^+ | \phi > + \sum_{m_1} \frac{< \phi | P_I^+ | m_1 >< m_1 | P_I^+ | \phi >}{(P_0^+ - P_{m_1}^+ + i\epsilon)}$$

$$+ \sum_{m_1} \frac{< \phi | P_I^- | m_1 >< m_1 | P_I^+ | \phi >}{(P_0^+ - P_{m_1}^+ + i\epsilon)} \tag{4.7}$$

Then the mass renormalization becomes

$$\delta M^2 = P_0^+ < \phi | P_I^- | \phi > + P_0^+ \sum_{n_1} \frac{< \phi | P_I^- | n_1 >< n_1 | P_I^- | \phi >}{(P_0^- - P_{n_1}^- + i\epsilon)}$$

$$+ P_0^+ \sum_{n_1} \frac{< \phi | P_I^- | n_1 >< n_1 | P_I^+ | \phi >}{(P_0^+ - P_{n_1}^+ + i\epsilon)}$$

$$+P_0^- < \phi|P_I^+|\phi > +P_0^- \sum_{m_1} \frac{< \phi|P_I^+|m_1 >< m_1|P_I^+|\phi >}{(P_0^+ - P_{m_1}^+ + i\epsilon)}$$

$$+P_0^- \sum_{m_1} \frac{< \phi|P_I^-|m_1 >< m_1|P_I^+|\phi >}{(P_0^+ - P_{m_1}^+ + i\epsilon)} \qquad (4.8)$$

We can also see now that the wave-function renormalization is

$$Z_\phi^{-1} = 1 - (\frac{\partial \delta P^-}{\partial P_0^-})_{p^2=m^2} - (\frac{\partial \delta P^+}{\partial P_0^+})_{p^2=m^2} \qquad (4.9)$$

As for coupling constant renormalization, we just take the operator $(P_I^- + P_I^+)$ between different initial, final states. We get

$$< \Psi_F|(P_I^- + P_I^+)|\Psi_I >= Z_e(Z_I Z_F)^{1/2}\{ < \phi_f|P_I^-|\phi_i > + < \phi_f|P_I^+|\phi_i >$$

$$+\sum_{n_1} \frac{< \phi_f|P_I^-|n_1 >< n_1|(P_I^- + P_I^+)|\phi_i >}{(P_0^- - P_{n_1}^- + i\epsilon)} + \sum_{m_1} \frac{< \phi_f|P_I^+|m_1 >< m_1|P_I^- + P_I^+|\phi_i >}{(P_0^+ - P_{m_1}^+ + i\epsilon)}$$

$$+ \sum_{n_1,n_2} \frac{< \phi_f|P_I^-|n_1 >< n_1|P_I^-|n_2 >< n_2|(P_I^- + P_I^+)|\phi_i >}{(P_0^- - P_{n_1}^- + i\epsilon)(P_0^- - P_{n_2}^- + i\epsilon)}$$

$$+ \sum_{m_1,m_2} \frac{< \phi_f|P_I^+|m_1 >< m_1|P_I^+|m_2 >< m_2|(P_I^- + P_I^+)|\phi_i >}{(P_0^+ - P_{m_1}^+ + i\epsilon)(P_0^+ - P_{m_2}^+ + i\epsilon)}$$

$$+ \sum_{n_1,m_2} \frac{< \phi_f|P_I^-|n_1 >< n_1|P_I^-|m_2 >< m_2|(P_I^- + P_I^+)|\phi_i >}{(P_0^- - P_{n_1}^- + i\epsilon)(P_0^+ - P_{m_2}^+ + i\epsilon)}$$

$$+ \sum_{m_1,n_2} \frac{< \phi_f|P_I^+|m_1 >< m_1|P_I^-|n_2 >< n_2|(P_I^- + P_I^+)|\phi_i >}{(P_0^+ - P_{m_1}^+ + i\epsilon)(P_0^- - P_{n_2}^- + i\epsilon)}\} \qquad (4.10)$$

5. One-Loop Fermion Mass Renormalization and Wave-Function Renormalization

We are ready to apply this to perturbation theory: we can calculate the fermion mass renormalization and wave-function renormalization to one-loop. This is pretty straight-forward.

Since both P^+ and P^- are dynamical, we get contributions from both of them. First, let us compute the contributions from P^-, denoted by δP^-, for an electron with momentum p :

$$\delta P^- = \delta P_1^- + \delta P_2^- + \delta P_3^- \tag{5.1}$$

where

$$\delta P_1^- = (-e)^2 \int \frac{d^2 k_\perp}{16\pi^3} dk^+ \frac{\theta(k^+)\theta(p^+ - k^+)}{[k^+]} \frac{1}{4} (\Gamma^i_{q0}(p, p-k, k) + \Gamma^i_{q0}(p-k, p, k))^2$$

$$\frac{1}{p^- - k^- - (p-k)^-} \tag{5.2}$$

which is due to the emission and reabsorbtion of a virtual photon ,coming from the iteration of $\mathcal{H}_{e^+e^-A}$.

The second piece comes from the four lepton piece of the Hamiltonian, $\mathcal{H}_{e^+e^-e^+e^-}$, where two of the fermions have been 'tied' together

$$\delta P_2^- = 2e^2 \int \frac{d^2 k_\perp}{16\pi^3} dk^+ \theta(k^+) \{ \frac{1}{[p^+ - k^+]^2} - \frac{1}{[p^+ + k^+]^2} \} \tag{5.3}$$

The last piece comes from the two-photon piece of the Hamiltonian, $\mathcal{H}_{e^+e^-AA}$, where the photon lines have been 'tied' together

$$\delta P_3^- = e^2 \int \frac{d^2k_\perp}{16\pi^3} \frac{dk^+\theta(k^+)}{[k^+]} \frac{\sigma_i \sigma^i}{[p^+ - k^+]} \tag{5.4}$$

The first contribution becomes, after introducing the parameters $[x] = \frac{[k^+]}{p^+}$ and $\kappa_\perp = k_\perp - xp_\perp$:

$$\delta P_1^- = \frac{e^2}{p^+} \int \frac{d(\kappa^2)}{16\pi^2} \int_0^1 dx \frac{1}{[1-x]} \frac{\kappa^2(\frac{4}{[x]^2} - \frac{4}{[x]} + 2)}{x(1-x)p^2 - \kappa^2 - xm^2} \tag{5.5}$$

We perform now transverse dimensional regularization, the most consistent regularization on light-front [15] and obtain

$$\delta P_1^- = -\frac{e^2}{16\pi^2} \{ \frac{p^2 - m^2}{p^+} [\frac{1}{\epsilon_t}(2ln\frac{p^+}{\lambda} - \frac{3}{2}) - \int_0^1 dx (\frac{2}{[x]} - 2 + x) ln(x\frac{m^2}{\mu^2} - x(1-x)\frac{p^2}{\mu^2})]$$

$$-2\frac{p^2}{p^+}[\frac{3}{2\epsilon_t} - \int_0^1 dx (2-x) ln(x\frac{m^2}{\mu^2} - x(1-x)\frac{p^2}{\mu^2})]\} \tag{5.6}$$

where $\epsilon_t = 1 - \frac{d_t}{2}$ (so that $\frac{1}{\epsilon_t}$ is equivalent to $ln\frac{\Lambda_\perp^2}{m^2}$ in ultraviolet cut-off regularization), λ is the infra-red regularization and μ is a mass scale introduced by dimensional regularization [8].

The second contribution, δP_2^- becomes

$$\delta P_2^- = \frac{2e^2}{p^+} \int \frac{d^2k_\perp}{8\pi^2} \int_0^\infty dx (\frac{1}{[1-x]^2} - \frac{1}{(1+x)^2}) \tag{5.7}$$

Using transverse dimensional regularization, this is just zero:

$$\delta P_2^- = 0 \tag{5.8}$$

The last contribution, δP_3^- becomes

$$\delta P_3^- = \frac{e^2}{p^+} \int \frac{d^2 k_\perp}{8\pi^2} \int_0^\infty \frac{dx}{[x]} \frac{2}{[1-x]} \tag{5.9}$$

Using transverse dimensional regularization, this is also zero:

$$\delta P_3^- = 0 \tag{5.10}$$

The total contribution to δP^- is

$$\delta P^- = -\frac{e^2}{16\pi^2} \{ \frac{p^2 - m^2}{p^+} [\frac{1}{\epsilon_t}(2ln\frac{p^+}{\lambda} - \frac{3}{2}) - \int_0^1 dx (\frac{2}{[x]} - 2 + x) ln(x\frac{m^2}{\mu^2} - x(1-x)\frac{p^2}{\mu^2})]$$

$$-2\frac{p^2}{p^+}[\frac{3}{2\epsilon_t} - \int_0^1 dx (2-x) ln(x\frac{m^2}{\mu^2} - x(1-x)\frac{p^2}{\mu^2})] \} \tag{5.11}$$

We compute now easily the contributions from δP^+, by using the explicit parity symmetry of our formalism:

$$\delta P_1^+ = -\frac{e^2}{16\pi^2} \{ \frac{p^2 - m^2}{p^-} [\frac{1}{\epsilon_t}(2ln\frac{p^-}{\lambda} - \frac{3}{2}) - \int_0^1 dx (\frac{2}{[x]} - 2 + x) ln(x\frac{m^2}{\mu^2} - x(1-x)\frac{p^2}{\mu^2})]$$

$$-2\frac{p^2}{p^-}[\frac{3}{2\epsilon_t} - \int_0^1 dx (2-x) ln(x\frac{m^2}{\mu^2} - x(1-x)\frac{p^2}{\mu^2})] \} \tag{5.12}$$

with λ and ϵ_t as above.

For the second piece δP_2^+ we get

$$\delta P_2^+ = \frac{e^2}{p^-}\int \frac{d^2k_\perp}{8\pi^2}\int\limits_0^\infty dy(\frac{2}{[1-y]^2} - \frac{2}{(1+y)^2}) \tag{5.13}$$

Using transverse dimensional regularization, this is zero ,as above:

$$\delta P_2^+ = 0 \tag{5.14}$$

We also get $\delta P_3^+ = 0$.

The total contribution to δP^+ is

$$\delta P^+ = -\frac{e^2}{16\pi^2}\{\frac{p^2-m^2}{p^-}[\frac{1}{\epsilon_t}(2ln\frac{p^-}{\lambda} - \frac{3}{2}) - \int\limits_0^1 dx(\frac{2}{[x]} - 2 + x)ln(x\frac{m^2}{\mu^2} - x(1-x)\frac{p^2}{\mu^2})]$$

$$-2\frac{p^2}{p^-}[\frac{3}{2\epsilon_t} - \int\limits_0^1 dx(2-x)ln(x\frac{m^2}{\mu^2} - x(1-x)\frac{p^2}{\mu^2})]\} \tag{5.15}$$

Putting all of this together, we get

$$\delta M^2 = p^+(\delta P^-)_{p^2=p^+p^-=m^2} + p^-(\delta P^+)_{p^2=p^+p^-=m^2}$$

$$= \frac{3e^2}{8\pi^2}m^2\frac{1}{\epsilon_t} \tag{5.16}$$

which is identical to the usual canonical quantization result [16] . The key is that we get the same singularity structure. Note the feature that infrared singularities cancel and ultraviolet singularities are only logarithmic, like in the canonical treatment. We also get the same coefficient as in the canonical calculation.

We can also obtain now the fermion wave-function renormalization[(4.9)]. It is

$$Z_2^{-1} = 1 - (\frac{\partial \delta P^-}{\partial p^-})_{p^2=p^+p^-=m^2} - (\frac{\partial \delta P^+}{\partial p^+})_{p^2=p^+p^-=m^2}$$

$$Z_2^{-1} = 1 - \frac{e^2}{4\pi^2}\{\frac{1}{\epsilon_t}(3 - ln\frac{m^2}{\lambda^2}) - ln\frac{m^2}{\lambda^2} ln\frac{\mu^2}{m^2}\} \tag{5.17}$$

Again, this is very similar to the usual calculations [16], down to the singularity structure, but with different coefficients than usual. The main difference is that we have products of infra-red and ultra-violet divergences, similar to [8].

6. One-Loop Photon Mass Renormalization and Wave-Function Renormalization

We look now at the one-loop quantum correction to the photon mass and wave-function . We proceed like we did in the previous chapter. We get one contribution from the remaining piece of the Hamiltonian $\mathcal{H}_{e^+e^-A}$ plus a contribution from the piece $\mathcal{H}_{e^+e^-AA}$:

$$\delta P^- = \delta P_1^- + \delta P_2^- \tag{6.1}$$

For a photon of energy q we get the following from the iteration of $\mathcal{H}_{e^+e^-A}$ (the overall minus sign comes from the fermion loop):

$$\delta P_1^- = (-1)\frac{(-2e)^2}{q^+}\int \frac{d^2k_\perp}{16\pi^3} dk^+ \theta(k^+)\theta(q^+ - k^+) Tr\{\chi^\dagger \chi \frac{q_i}{q^+}\frac{q^i}{q^+}\chi^\dagger \chi \frac{1}{q^- - k^- - (q-k)^-}\} \tag{6.2}$$

which becomes, upon using $[x] = \frac{[k^+]}{q^+}$ and $\kappa_\perp = k_\perp - xq_\perp$

$$\delta P_1^- = -\frac{4e^2}{q^+}\int \frac{d(\kappa^2)}{16\pi^2}\int_0^1 dx \frac{(1-x)xq_\perp^2}{x(1-x)q^2 - \kappa^2 - m^2} \tag{6.3}$$

Using transverse dimensional regularization, we get

$$\delta P_1^- = \frac{e^2}{16\pi^2}\frac{q^+q^- - q^2}{q^+}\frac{4}{6\epsilon_t} \tag{6.4}$$

where the meaning of ϵ_t and λ is as before.

For the second contribution, we have as follows after tying together $\mathcal{H}_{e^+e^-AA}$:

$$\delta P_2^- = e^2 \int \frac{d^2k_\perp}{16\pi^3}\frac{dk^+\theta(k^+)}{[k^+]}\frac{\sigma_i\sigma^i}{[p^+ - k^+]} \tag{6.5}$$

which becomes, upon using $[x] = \frac{[k^+]}{q^+}$ and $\kappa_\perp = k_\perp - xq_\perp$

$$\delta P_2^- = \frac{e^2}{q^+}\int \frac{d^2k_\perp}{8\pi^2}\int_0^\infty \frac{dx}{[x]}\frac{2}{[1-x]} \tag{6.6}$$

Using transverse dimensional regularization, we get $\delta P_2^- = 0$.

The total contribution to δP^- is

$$\delta P^- = \frac{e^2}{24\pi^2}\frac{q^+q^- - q^2}{q^+}\frac{1}{\epsilon_t} \tag{6.7}$$

We get a similar result for δP^+'s :

$$\delta P_1^+ = \frac{e^2}{24\pi^2}\frac{q^-q^+ - q^2}{q^-}\frac{1}{\epsilon_t} \tag{6.8}$$

plus $\delta P_2^+ = 0$ also.

The correction to the photon mass becomes then

$$\delta\mu^2 = q^+(\delta P^-)_{q^2=q^+q^-=0} + q^-(\delta P^+)_{q^2=q^+q^-=0}$$

$$\delta\mu^2 = 0 \tag{6.9}$$

Note that the one-loop correction to the photon mass is exactly zero!

This is the first front-form calculation which gives a zero photon mass renormalization (to one loop). It is reassuring that we get this result, since we quantized only the physical degrees of freedom.

We get now the photon wave-function renormalization by simply using the formula introduced above [(4.9)] :

$$Z_3^{-1} = 1 - (\frac{\partial \delta P^-}{\partial q^-})_{q^2=0} - (\frac{\partial \delta P^+}{\partial q^+})_{q^2=0}$$

$$Z_3^{-1} = 1 + \frac{e^2}{12\pi^2}\frac{1}{\epsilon_t} \tag{6.10}$$

This also agrees with the usual canonical results [16] , and insures that we get the same beta-function for the photon as well .

7. Ward Identity

There is one more thing to compute and that is the vertex correction. We also want to check that the vertex renormalization Z_1 is the same as the one for the fermion, Z_2. For this we use the definition in [(4.10)], with the initial, final states being electrons, with momenta p and p' respectively, and the vertex being one photon with momentum q . We obtain , in the limit in which q, the photon momentum transfer $q = p' - p = 0$

$$(\frac{2eq^i}{q^+} + \frac{2eq^i}{q^-}) = Z_1 Z_2 (\frac{2eq^i}{q^+} + \frac{2eq^i}{q^-}) \times$$

$$\{1 + 2e^2 \int \frac{d^2\kappa}{16\pi^2} \int_0^1 dx \frac{1}{[1-x]} \frac{x(1-x)\kappa^2(\frac{4}{[x]^2} - \frac{4}{[x]} + 2)}{x(1-x)p^2 - \kappa^2 - xm^2}\} \tag{7.1}$$

where the factor of two in front of the integral comes from the two time orderings allowed in this case. We immediately see that this is just

$$Z_1^{-1}Z_2^{-1} = \{1 - 2(\frac{\partial \delta P^-}{\partial p^-})_{p^2=p^+p^-=m^2} - 2(\frac{\partial \delta P^+}{\partial p^+})_{p^2=p^+p^-=m^2}\} \tag{7.2}$$

In perturbation theory , where the coupling $\alpha = \frac{e^2}{4\pi}$ is small, we can approximate the right-hand side as the square of Z_2^{-1}. Thus, we get the Ward identity

$$Z_1^{-1}Z_2^{-1} = (Z_1^{-1})^2, Z_1^{-1} = Z_2^{-1} \tag{7.3}$$

8. Conclusions

This new method of quantizing fields on the front-form seem to give consistent results in perturbation theory, applied to QED. It seems that the singularity structure is simplified, so that to one-loop the infrared and ultraviolet singularities are logarithmic only. We also obtain that the photon mass stays zero to one-loop, a first for front-form quantization.

9. Appendix A

Using the definition of commutation relations in the interaction representation like Harindranath and Zhang [8], we have

$$\partial^- A_P^i(x) = \frac{1}{i}[A_P^i, P_0^-] = \frac{1}{2}(\frac{1}{\partial^+})\partial_i^2 A_P^i(x^-, x_\perp) \tag{9.1}$$

Now notice the following subtle point : we can add

$$J(x^-, x^+, x_\perp) = \frac{1}{2}(\frac{1}{\partial^+})[\partial_i^2 A_M^i(x^+, x_\perp) - \partial_i^2 B^i(x_\perp)] \tag{9.2}$$

since this is trivially zero due to the anti-symmetry of $(\partial^+)^{-1}$, and due to the fact that neither A^i_M nor B^i have any x^- dependence! So we obtain

$$\partial^- A^i_P(x) = J(x^+, x^-, x_\perp) + \frac{1}{2}(\frac{1}{\partial^+})\partial_i^2 A^i_P(x^-, x_\perp) = \frac{1}{2}(\frac{1}{\partial^+})\partial_i^2 A^i(x^-, x_\perp) \quad (9.3)$$

We obtain a similar result for A^i_M. Then, putting it all together, we get the usual dispersion relation

$$\partial^+\partial^- A^i(x) = \partial^+\partial^-[A^i_M(x) + A^i_P(x) - B^i(x_\perp)] = \partial_j^2 A^i(x) \quad (9.4)$$

since $\partial^+\partial^- B^i(x_\perp) = 0$ and each piece, A^i_P and A^i_M contributes $\frac{1}{2}$ to the final result!

Likewise, for the fermionic degrees of freedom, we get

$$\partial^- \psi_+(x) = \frac{1}{i}[\psi_+, P_0^-] = \frac{1}{2\partial^+}(\partial_j^2 - m^2)\psi_+(x) \quad (9.5)$$

As in the vector field case, we can add the following (zero!) term

$$K_1(x^+, x^-, x_\perp) = \frac{1}{2\partial^+}(\partial_j^2 - m^2)\psi_-(x) \quad (9.6)$$

and obtain

$$\partial^- \psi_+(x) = \frac{1}{2\partial^+}(\partial_j^2 - m^2)\psi(x) \quad (9.7)$$

Similarly for ψ_- we get

$$\partial^+ \psi_-(x) = \frac{1}{i}[\psi_-^\dagger, P_0^+] = \frac{1}{2\partial^+}(\partial_j^2 - m^2)\psi(x) \quad (9.8)$$

after we have added

$$K_2(x^+, x^-, x_\perp) = \frac{1}{2\partial^-}(\partial_j^2 - m^2)\psi_+(x) \quad (9.9)$$

Note that here, since both ψ_+ and ψ_- are independent, we get two equations of motions. Putting them together, we get

$$\partial^{+}\partial^{-}\psi(x) = (\partial_j^2 - m^2)\psi(x) \tag{9.10}$$

so we see that the 1/2's add up to give the full equation of motion for fermion field.

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