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Relation between Quantization on Null-Surfaces and the Characteristic Initial Value Problem*

OVID C. JACOB†‡

Stanford Linear Accelerator Center

Stanford University, Stanford, California 94309

ABSTRACT

We consider quantization along null-surfaces. We note that null-surfaces are characteristic surfaces, which requires a different treatment than the usual equal-time Cauchy initial data quantization. We examine this for quantum field theories and show that by considering quantization on null-surfaces as a characterisitc initial value problem, we can avoid initial relations - constraints - between the fields. We are also able to show that the Green's function obtained is identical in form to that for equal-time quantization.

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† jacob@slacvm.slac.stanford.edu
‡ Permanent Address: *Department of Physics and Astronomy 1801 East Cotati Avenue Sonoma State University Rohnert Park, CA 94928*

1. Introduction

In the past year or so there have been several proposals which have re-examined how to quantize fields on the null-surfaces [1] [2] [3] [4] [5] [6] . The work of the present author and that of Heinzl and Werner have looked at the possibility that we need to take a fresh new approach to the quantization of fields on the null-surface, quite different than what has been done in the past, whether done by Dirac [7] ; Weinberg, who called it the infinite-momentum frame (IMF) [8] ; the null-surfaces coordinate method of Chang and Ma or Kogut and Soper [9] [10] [11] (the recent work of Ligterink and Bakker is very similar to that of Chang, Ma and Kogut, Soper [9], [10]) [12] ; of that of discretized null-surfaces quantization of Brodsky and co-workers [13] [14] [15] ; or that of reduced phase-space null-surfaces quantization [6]. These approaches are actually very similar to the canonical method with the early ones clinging rather close to the manifest covariant method (this is short for **Lorentz** manifest covariance, which I will imply throughout, whenever I use this phrase) [7], [8], [9], [10], the more recent ones straying somewhat [13], [6].

In this work we want to show that the methods proposed recently by the present author and that of Heinzl and Werner [5] are very different that these previous approaches. We hope to convince the reader that these methods have the highest probability of succeeding in constructing a consistent theory of quantum fields on the null-surface because the present method is informed by the classical characteristic initial value problem.

The point of view we take in this paper is that quantization is clearly related to the initial value problem. We keep track of the degrees of freedom as we relate the classical equations to the quantum fields, or as we re-interpret the classical field solutions as quantum fields. The latter operation is trivial and almost unnoticed in the case where we are dealing only with independent degrees of freedom. It becomes rather subtle and involved when **dependent** degrees of freedom come in. The advantage of the present method is that the initial data is consistent with the characteristic initial value problem as studied in classical gravity [16] . Also, it seems that the present method involves only **independent** degrees of freedom, so the re-interpretation of classical solutions as quantum fields is greatly simplified.

This paper is organized as follows: first, we discuss the manifest covariant method of quantization as a Cauchy data initial value problem. By this we mean the usual equal-time quantization on equal $t = const.$ hyperplanes. We then discuss briefly some of the other methods of quantizing on the null-surfaces and point out some of their shortcomings Then, we discuss the null-surfaces quantization method and show that this method of quantization is a characteristic initial value problem, with Dirichlet initial data, which forces us to give initial data for the fields on surfaces $\Sigma_+ = \{u = x^+ \geq 0\}$, $\Sigma_- = \{v = x^- \geq 0\}$, $\Sigma_0 = \{u = v = 0\}$.

2. Manifest Covariant Quantization as a Cauchy Initial Value Problem

Consider for simplicity a scalar field ϕ obeying the Klein-Gordon equation

$$\partial_{tt}\phi - \partial_{xx}\phi - \partial_{yy}\phi - \partial_{zz}\phi = F(\phi) \tag{2.1}$$

where $\partial_\alpha = \frac{\partial}{\partial x^\alpha}$ for any symbol α. Repeated symbols mean two , three, etc. number of partial derivatives. $F(\phi)$ is an arbitrary function of the field ϕ. In this work we are only considering scalar fields interacting amongst each other. It is rather straight-forward to extend the present discussion to other fields , like fermions and vectors. The main features will be clear at this level.

In the usual classical approach, to specify a unique solution, we need the field as well as the first derivative at some time t_0: $\phi(t_0, \overrightarrow{x})$, $\partial_{t_0}\phi(t_0, \overrightarrow{x})$. This time is may be taken to be $t_0 = 0$:

$$f(\overrightarrow{x}) = \phi(t_0, \overrightarrow{x})_{t_0=0}, g(\overrightarrow{x}) = \partial_{t_0}\phi(t_0, \overrightarrow{x})_{t_0=0} \tag{2.2}$$

With this initial Cauchy data , plus the equation of motion [(2.1)], we can easily develop a local solution for the field ϕ in terms of a Taylor series around $t = 0$ written in terms of the initial data:

$$\phi(t, \overrightarrow{x}) = f(\overrightarrow{x}) + tg(\overrightarrow{x}) + \frac{t^2}{2!}\partial_{tt}\phi(t, \overrightarrow{x})_{t=0} + \frac{t^3}{3!}\partial_{ttt}\phi(t, \overrightarrow{x})_{t=0} + ... \tag{2.3}$$

The first two terms are the given initial data. The third term, $\partial_{tt}\phi(t, \overrightarrow{x})_{t=0}$ can be easily obtained from the equation of motion:

$$\partial_{tt}\phi(t, \overrightarrow{x}) = \partial_{xx}f(\overrightarrow{x}) + \partial_{yy}f(\overrightarrow{x}) + \partial_{zz}f(\overrightarrow{x}) + F(f(\overrightarrow{x})) \tag{2.4}$$

Likewise, higher order terms can be obtained by use of the equation of motion and the initial data. To show one more example,

$$\partial_{ttt}\phi(t, \overrightarrow{x}) = \partial_{xx}g(\overrightarrow{x}) + \partial_{yy}g(\overrightarrow{x}) + \partial_{zz}g(\overrightarrow{x}) + \frac{\partial F(f(\overrightarrow{x}))}{\partial\phi}g(\overrightarrow{x}) \tag{2.5}$$

Thus, we are able to construct the whole Taylor series for the **local** solution of the field out of the initial data and the equation of motion. To obtain the **global** solution everywhere is a more difficult question and depends on the topology of the manifold on which we are working. I will not address these issues here. (I have looked at some of these issues in a recent paper on zero modes on null-surfaces [17] .)

But there is another way of looking at this. Ever since Green's papers in potential theory [18] , we have been able to recast differential equations with initial value boundary conditions into integral equations with Green's functions, which are defined to contain the boundary conditions of initial value data in them [19] .

Both in the classical case as in the quantum case, these Green's functions are distributions. In the classical case, though, the fields themselves remain regular functions, while in the quantum case, the fields become operator-valued functionals. In both cases, the initial values of boundary conditions become definitions (or "prescriptions") treating the singularities [20] . Constructing the Green's function is the program of constructing a kernel to be used in propagating or extending the initial field over the given manifold [21] .

In practice there are few exact solutions, and we almost always fall back on perturbative solution of the following equation [22] :

$$\phi(t, \overrightarrow{x}) = \int_S dA\{\phi(y)\partial_n\Delta(x-y) - \Delta(x-y)\partial_n\phi(y)\} + \int_V dV F(\phi(y))G_0(x-y) \quad (2.6)$$

where F is the inhomogeneity, integrated over the whole volume V, G_0 is the Green's function for the homogeneous equation and ∂_n is the normal to the initializing surface S. The distribution $\Delta(x-y)$ satisfies the equation:

$$(\partial_{tt} - \partial_{xx} - \partial_{yy} - \partial_{zz})\Delta(x-y) = 0 \quad (2.7)$$

and at the point $x^0 = y^0$, $\partial_{x^0}\Delta(x-y) = -\delta^3(\overrightarrow{x} - \overrightarrow{y})$, The first term in equation [(2.6)] corresponds to Itzykson and Zuber's $\phi^{(0)}$, the solution to the homogeneous equation, so that ϕ satisfies the boundary(initial value) conditions. Thus, we construct the Green's function for the free theory, but then we need to know the full solution at one point y to propagate it to the point x in the second term above.

An alternative construction is to obtain the Green's function for the inhomogeneous equation, and thus forget about the difficult first term. But now, we have just shifted the problem to constructing the "full" Green's function, which is equally hard. In most cases, we can solve for the Green's function only approximately, using a perturbative expansion.

In this sense, quantization can be viewed as a question of extending initial data over manifolds. It may be viewed as the Feynman view or the propagator view or the path integral view [21].

Let us look now at the interacting quantum field case. The paper by Heinzl and Werner [5] has a clear discussion of manifest covariant quantization as a Cauchy initial value problem. They point out that the field ϕ needs two pieces of input, the field at some time t and the derivative of the field at the same time, as well as the distribution $\Delta(x-y)$ obtained from the equation of motion for the field ϕ :

$$\phi(t,\overrightarrow{x}) = \int d^3y\{f(\overrightarrow{y})\partial_t\Delta(x-y) - \Delta(x-y)g(\overrightarrow{y})\} + \int_V dV F(\phi(y))G_0(x-y) \tag{2.8}$$

Thus, we need as initial data, both the field ϕ and its derivative $\partial_t\phi$ to specify the quantum field locally at arbitrary space-time points. With this construction we get back the initial data $\phi(0,\overrightarrow{x}) = f(\overrightarrow{x})$.

Of course, there is a problem with this construction: it gives the variable t a privileged position in defining an initial surface $t = const$. A step away from this specificity is to notice that the equation [(2.8)] is just the expression for the full or exact Green's theorem as seen, for instance in Jackson [19]!

$$\phi(t,\overrightarrow{x}) = \int\limits_S dA\{\phi(y)\partial_n\Delta(x-y)-\Delta(x-y)\partial_n\phi(y)\}+\int\limits_V dV F(\phi(y))G_0(x-y) \quad (2.9)$$

We see now that our surface S is $S = \{t = 0\}$, our area differential dA is just d^3y, and out normal derivative ∂_n is ∂_t.

Thus, the initial value problem implies that we define the field ϕ in terms of the Green's function such that we obtain the usual commutator, like Werner and Heinzl [5] in the free case:

$$[\phi(x),\phi(0)] = i\Delta(x) \quad (2.10)$$

In the general case we obtain

$$[\phi(x),\phi(0)] = \int dy^1 d^2y_\perp\{[\partial_{y^0}\phi(y),\phi(0)]\Delta(x-y) - [\phi(y),\phi(0)]\partial_{y^0}\Delta(x-y)\}_{y^0=0}$$

$$+\int\limits_V dV[F(\phi(y)),\phi(0)]G_0(x-y) \quad (2.11)$$

Let us point out that for $F = 0$, we get the free case, as expected.

3. Null-Surfaces Quantization

While it is true that the Cauchy data on characteristic surfaces is ill-posed, nonetheless, Dirichlet data (the value of the field) is quite well-posed on **two** surfaces, as Müller zum Hagen and Seifert have pointed out [16]. The case C in their paper is essentially our case. [23] . I won't repeat the arguments of zum Hagen and Seifert here, which I think are most convincing. Rather, I wish to show that giving the initial data on the surfaces $\Sigma_+\{u = x^+ \geq 0\}$, $\Sigma_-\{v = x^- \geq 0\}$, $\Sigma_0\{u = v = 0\}$, as mentioned in the Introduction, is equivalent to giving Cauchy data on $t = 0$. Let us look first at the classical scalar field.

Consider the Klein-Gordon equation for the scalar field as before, with the initial data $\phi(u, v = 0, x_\perp) = f(u, x_\perp)$, $\phi(u = 0, v, x_\perp) = g(v, x_\perp)$ and $\phi(u = 0, v = 0, x_\perp) = \alpha(x_\perp)$. In these new coordinates, $x^\mu = (u, v, x, y) = (u, v, x_\perp)$ the Klein-Gordon equation of motion becomes:

$$\partial_{uv}\phi - \partial_{xx}\phi - \partial_{yy}\phi = F(\phi) \tag{3.1}$$

As it was pointed out by Courant and Hilbert [24] , the characteristic surface for the wave equation is given by the hypersurfaces $u = 0$ and $v = 0$.

The general solution, locally, is given by a Taylor series about $u = 0$:

$$\phi_P(u, v, x_\perp) = g(v, x_\perp)$$

$$+u\partial_u\phi(u, v, x_\perp)_{u=0} + \frac{u^2}{2!}\partial_{uu}\phi(u, v, x_\perp)_{u=0} + ... \tag{3.2}$$

It can also be written as an expansion around $v = 0$:

$$\phi_M(u, v, x_\perp) = f(u, x_\perp)$$

$$+v\partial_v\phi(u, v, x_\perp)_{v=0} + \frac{v^2}{2!}\partial_{vv}\phi(u, v, x_\perp)_{v=0} + .. \tag{3.3}$$

Finally, we have an expansion about the point $u = v = 0$:

$$\phi_0(u, v, x_\perp) = \alpha(x_\perp)$$

$$+uv\partial_{uv}\phi(u,v,x_\perp)_{u=v=0}+\frac{(uv)^2}{2!2!}\partial_{uvuv}\phi(u,v,x_\perp)_{u=v=0}+.. \qquad (3.4)$$

We will take for our general solution the following form:

$$\phi(u,v,x_\perp)=\phi_P(u,v,x_\perp)+\phi_M(u,v,x_\perp)-\phi_0(u,v,x_\perp) \qquad (3.5)$$

with hindsight of what we want the quantum field to look like [4]. So far, all three expansions seem equivalent, so we have no reason to prefer one over the other.

We need now the u- and v-derivatives of the field ϕ in terms of the given data. For the u derivative we can re-write the equation of motion thus

$$\frac{\partial}{\partial v}(\partial_u\phi)=\partial_{xx}\phi+\partial_{yy}\phi+F(\phi) \qquad (3.6)$$

This gives the u-derivative on $u=0$:

$$\partial_u\phi=\int^v dv^1\{\partial_{xx}g(v^1,x_\perp)+\partial_{yy}g(v^1,x_\perp)+F(g(v^1,x_\perp))\} \qquad (3.7)$$

Likewise, we get the v-derivative on $v=0$:

$$\partial_v\phi=\int^u du^1\{\partial_{xx}f(u^1,x_\perp)+\partial_{yy}f(u^1,x_\perp)+F(f(u^1,x_\perp))\} \qquad (3.8)$$

To obtain the second order u- and v-derivative, observe that we can write the u-derivative of the equation of motion

$$\partial_{uu}\partial_v\phi=\partial_{xx}\partial_u\phi+\partial_{yy}\partial_u\phi+\frac{\partial F(\phi)}{\partial\phi}\partial_u\phi \qquad (3.9)$$

where the mixed derivative is

$$\partial_{xx}\partial_u\phi=\int dv^1\{\partial_{xxxx}\phi+\partial_{xxyy}\phi+[\frac{\partial^2F(\phi)}{\partial^2\phi}(\partial_x\phi)^2+\frac{\partial F(\phi)}{\partial\phi}\partial_{xx}\phi]\} \qquad (3.10)$$

so the second u-derivative on $u = 0$ becomes:

$$\partial_{uu}\phi = \int^{v} dv^1 \{ \int^{v^1} dv^2 \{ \partial_{xxxx} g(v^2, x_\perp) + \partial_{xxyy} g(v^2, x_\perp) + [\frac{\partial^2 F(g)}{\partial^2 g}(\partial_x g)^2 + \frac{\partial F(g)}{\partial g}\partial_{xx} g(v^2, x_\perp)] \}$$

$$+ \int^{v^1} dv^2 \{ \partial_{yyxx} g(v^2, x_\perp) + \partial_{yyyy} g(v^2, x_\perp) + [\frac{\partial^2 F(g)}{\partial^2 \partial g}(\partial_x g)^2 + \frac{\partial F(g)}{\partial g}\partial_{yy} g(v^2, x_\perp)] \}$$

$$+ \int^{v} dv^1 \frac{\partial F(g)}{\partial g} \int^{v^1} dv^2 [\partial_{xx} g(v^2, x_\perp) + \partial_{yy} g(v^2, x_\perp) + F(g(v^2, x_\perp))] \qquad (3.11)$$

We get a similar expression for the second v-derivative, with v replaced by u and g replaced by f.

Let us construct now the mixed derivatives on Σ_0. The first order is obtained just from the equation of motion:

$$\partial_{uv}\phi = \partial_{xx}\alpha(x_\perp) + \partial_{yy}\alpha(x_\perp) + F(\alpha(x_\perp)) \qquad (3.12)$$

We observe that we don't have mixed powers of derivatives of u or v on Σ_0 (like $\partial_{uu}\partial_v$) because this involves either $\partial_u\phi$ or $\partial_v\phi$ on Σ_0, which is zero by the definitions above.

Then the next higher order contribution is

$$\partial_{uvuv}\phi = [\partial_{xx} + \partial_{yy} + \frac{\partial F(\alpha)}{\partial \alpha}][\partial_{xx}\alpha + \partial_{yy}\alpha + F(\alpha)] \qquad (3.13)$$

It is clear now that we can build all higher order derivatives and thus obtain the Taylor series above [(3.3)]. Note that we need and use the information given on the surfaces Σ_+ , Σ_- as well as Σ_0. In this situation, this formulation is quite similar to the manifest covariant formulation [25] .

We look now at the quantum case, as studied by Heinzl and Werner [5]. We notice immediately that the form which they use for the field ϕ is like what we wrote for the classical theory above:

$$\phi(u,v,x_\perp) = \int_0^\infty du^1 d^2y_\perp \{ f(u^1,y_\perp)\frac{\partial}{\partial u^1}\Delta(x-y) - \Delta(x-y)\frac{\partial}{\partial u^1} f(u^1,y_\perp)\}$$

$$\int_0^\infty dv^1 d^2z_\perp \{ g(v^1,z_\perp)\frac{\partial}{\partial v^1}\Delta(x-y) - \Delta(x-y)\frac{\partial}{\partial v^1} g(v^1,z_\perp)\} - \phi(0,0,x_\perp)$$

$$\int_0^\infty du^1 \int_0^\infty dv^1 d^2y_\perp F(\phi(y))G_0(x-y) \tag{3.14}$$

where the distribution $\Delta(x-y)$ is obtained from the equation:

$$(\partial_{uv} - \partial_{xx} - \partial_{yy})\Delta(x-y) = 0 \tag{3.15}$$

and G_0 is the solution to the following equation:

$$(\partial_{uv} - \partial_{xx} - \partial_{yy})G_0(x-y) = \delta(u-u^1)\delta(v-v^1)\delta^2(x_\perp - y_\perp) \tag{3.16}$$

We have to add the third term in ϕ to avoid double counting at the wedge Σ_0, where $f(x_\perp) = g(x_\perp) = \alpha(x_\perp)$ [16], as well as to agree with Green's theorem, to be seen shortly. Notice that we have both the field on the two surfaces and its derivatives. The derivatives are such that we can take them easily: need $\partial_u f(u,x_\perp)$ on Σ_- and $\partial_v g(v,x_\perp)$ on Σ_+ , so these can be obtained easily.

As before, we still have the problem that we have singled out , now two variables, u and v. Like in the manifest covariant case, we can move away from this specificity by noting that we again get Green's theorem, but now with a new surface, and can write the same equation using Green's function as before!

$$\phi(u,v,x_\perp) = \int\limits_{S_{ns}} dA\{\phi(y)\partial_{n_{ns}}\Delta(x-y) - \Delta(x-y)\partial_{n_{ns}}\phi(y)\} + \int dV F(\phi(y))G_0(x-y) \tag{3.17}$$

Now, the surface S_{ns} is our initializing surface $S_{ns} = \{\Sigma_+, \Sigma_-, \Sigma_0\}$, the area differential is now $dA = \{dud^2y_\perp, dvd^2y_\perp, d^2y_\perp\}$ and the normal derivative is $\partial_{ns} = \{\partial_v, \partial_u, \partial^2_{y^2_\perp}\}$. The last term involves the null-surface Green's function G_0.

The first two terms in Green's integral can be seen easily. The third term is a bit trickier. The point is that $\Delta(0,0,x_\perp - y_\perp)$ is just the commutator of the fields on Σ_0, where the fields agree, hence they are equal. The commutator is then zero. The remaining piece is just $\partial^2_{y^2_\perp}\Delta(x-y) = \delta^2(x_\perp - y_\perp)$.

We see that this avoids the problem of double counting on Σ_0 and that it also agrees with the form presented in previous papers [1], [2] as well as with Green's theorem.

What about the commutator now? Like in Heinzl and Werner [5], this allows us to construct now the commutator $[\phi(x),\phi(0)]$, which for the free case is identical to their result. In the more general case, we obtain

$$[\phi(x),\phi(0)] = \int\limits_0^\infty du^1 d^2y_\perp\{[\phi(y),\phi(0)]\frac{\partial}{\partial u^1}\Delta(x-y) - \Delta(x-y)\frac{\partial}{\partial u^1}[\phi(y),\phi(0)]\}_{v=0}$$

$$+\int\limits_0^\infty dv^1 d^2z_\perp\{[\phi(y),\phi(0)]\frac{\partial}{\partial v^1}\Delta(x-y) - \Delta(x-y)\frac{\partial}{\partial v^1}[\phi(y),\phi(0)]\}_{u=0} - [\phi(x),\phi(0,0,x_\perp)]$$

$$+\int\limits_0^\infty du^1 \int\limits_0^\infty dv^1 d^2y_\perp [F(\phi(y)),\phi(0)]G_0(x-y) \tag{3.18}$$

We see that the term without the integral vanishes identically since the fields agree on Σ_0. We obtain the same relationship as Heinzl and Werner [5], eq. 3.15 for the case where $F = 0$. Furthuermore, we note that the limits of integration are precisely what the surfaces Σ_+ and Σ_- require: for $x^- = 0$, x^+ varies from 0 to ∞; likewise, for $x^+ = 0$, x^- varies from 0 to ∞.

4. Conclusions

We have studied the question of quantization as a characteristic initial value problem on null-surfaces. We found that the olde fashioned methods of IMF and the canonical null-surfaces quantization might have problems interpreting the classical solutions as unique quantum fields. The methods of Heinzl and Werner [5] and of the present author [3], [2], [1] seem most likely to give unique solutions and therefore unambiguous field theoretical interpretations. Furthermore, we find that our formulation agrees perfectly well with a Green's function formulation of quantum field theory.

There is a difference between this work and Heinzl and Werner: the latter choose to quantize on one surface only, eventhough they themselves remarked that data on **two** surfaces are needed - see discussion after equation 3.15 in [5].

We, on the other hand, have decided to take the point of view that data on the surfaces Σ_+, Σ_- and Σ_0 are needed. Note that the Figure 1 of Heinzl and Werner [5] is same as the surfaces we just mentioned! We have been able to apply this method so for to QED and QCD in [1] and [26] respectively.

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