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Null-Surfaces Quantization of Quantum ChromoDynamics in the Heavy Quark Limit : The Quantum ChromoDynamics Renormalization Constants[⋆]

Ovid C. Jacob[†‡]

Stanford Linear Accelerator Center

Stanford University, Stanford, California 94309

ABSTRACT

In this paper we study the QCD renormalization constants $Z_{QCD} = \{Z_Q, Z_q, Z_A, Z_g\}$ in the heavy quark limit of QCD. We compute these quantities using null-surfaces quantization, a method introduced recently to study gauge fields, as well as the perturbation approximation introduced in a previous work. We recover the standard covariant results for all the one-loop mass corrections and wave-function renormalizations. Specifically, we find that the massless fermion and gauge boson remains massless to one-loop.

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† jacob@slacvm.slac.stanford.edu
‡ Permanent Address: *Department of Physics and Astronomy 1801 East Cotati Avenue Sonoma State University Rohnert Park, CA 94928*

1. Introduction

In this paper we compute the QCD renormalization constants for heavy quark limit of QCD in null-surfaces quantization, which we have introduced recently [1] [2] [3] [4] (See also [5] as well as [6] for discussions on treating x^+ and x^- equally.) The main reason we study the heavy quark QCD sector is that in this region, with the large quark mass m_Q present we can justify the use of perturbation theory. Even so, strictly speaking, this perturbative expansion we obtain is not well defined as the asymptotic states QCD are hadrons rather than the quark and gluon fields we have in the present theory. Nonetheless, we think it is a worthwhile project to start developing heavy quark effective theory in this new null-surface quantization method.

This initial null-surfaces we employ are a bit unusual. Imagine that we take the plane $t = 0$ and we deform our surface about $z = 0$ so that we form a V-shaped surface making 45 degrees with the plane $t = 0$; or, if we introduce the coordinates $x^{\pm} = t \pm z$, our surface will be $\Sigma = \{\Sigma_+ = x^+ \geq 0, \Sigma_- = x^- \geq 0, \Sigma_0 = x^+ = x^- = 0\}$. See also figure 1, case C of Mulller zum Hagen and Seifert [7] . This is then our initial surface, made up of null-surfaces. Some questions regarding the Lorentz structure of this approach are discussed in more detail in Appendix A.

The method follows rather closely the previous work for Abelian gauge theory: we formulate the null-surface quantization along the two surfaces mentioned above, and follow Faddeev and Jackiw's method of reduced phase space quantization [8] as well as the methods of Harindranath and Zhang [9] . We find then that this allows us to formulate a consistent theory of quantum fields on light-cone (along null-surfaces). Our first such test was for QED [1], [3]. We want now to extend this approach to non-Abelian gauge theories.

The outline is as follows: we introduce the reduced phase quantization of QCD in the presence of two types of quarks: a massive (or heavy) quark of mass m_Q and a massless quark; we briefly discuss the quantization of the fields; we then display the Hamiltonians to be used in perturbation theory; we then compute the heavy and light quark one-loop mass and wave-function renormalization, and find agreement with the covariant calculations; finally, we compute the gluon one-loop mass and wave-function renormalization. As in the Abelian case, we find that the gauge boson mass remains zero to one-loop. We also find that the massless (light) quark also remains massless to one-loop.

2. Reduced Phase Space Quantization of Heavy Quark QCD

We follow the approach of previous work [1], [3], [9], to study QCD in the heavy quark limit, where we have a massive (heavy) quark Q with mass m_Q and a massless quark q :

$$\mathcal{L} = -\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\partial^\nu\overline{Q}\gamma_\nu Q - \overline{Q}\frac{i}{2}\partial^\nu\gamma_\nu Q - m_Q\overline{Q}Q + \frac{i}{2}\partial^\nu\overline{q}\gamma_\nu q - \overline{q}\frac{i}{2}\partial^\nu\gamma_\nu q + \mathcal{L}_I \quad (2.1)$$

where $\mathcal{L}_I$ is the interaction part of the Lagrangean:

$$\mathcal{L}_I = +g\overline{Q}\gamma^\nu Q A_\nu + g\overline{q}\gamma^\nu q A_\nu \quad (2.2)$$

In this case, we have $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\mu A^\nu - ig[A^\mu, A^\nu]$, $A^\mu = \sum_a A^\mu_a T^a$ is the gluon field, T^a are the generators for the color $SU(3)$ group, with $[T^a, T^b] = if^{abc}T^c$, f^{abc} being the $SU(3)$ color structure functions. We also have $Tr(T^aT^b) = T_F\delta^{ab} = \frac{1}{2}\delta^{ab}$.

Our null-surface coordinates are $x^{\pm} = x^0 \pm x^3$. The Euler-Lagrange equations of motion are

$$\partial_\mu F^{\mu\nu a} + g\overline{Q}\gamma^\nu T^a Q + g\overline{q}\gamma^\nu T^a q + g f^{abc} A_{\mu b} F_c^{\mu\nu} = 0 \tag{2.3}$$

$$(i\gamma_\mu \partial^\mu - m_Q + g\gamma_\mu A^\mu)Q = 0 \tag{2.4}$$

$$(i\gamma_\mu \partial^\mu + g\gamma_\mu A^\mu)q = 0 \tag{2.5}$$

To obtain the Hamiltonians for evolution along x^+ and x^-, we follow the approach of previous papers [9], [3] so we write $\mathcal{L}$ out explicitly :

$$\begin{aligned}
\mathcal{L} d^4x = \{ & -\frac{1}{2} F_{ija} F^{ija} - F_{+-a} F^{+-a} - F_{+ia} F^{+ia} - F_{-ia} F^{-ia} \\
& +2gQ_+^\dagger Q_+ A_+ + 2gQ_-^\dagger Q_- A_- - gQ_+^\dagger \gamma_0 \gamma_i A^i Q_- - gQ_-^\dagger \gamma_0 \gamma_i A^i Q_+ + Q_+^\dagger \frac{i\partial^-}{2} Q_+ - \frac{i}{2}(\partial^- Q_+^\dagger) Q_+ \\
& +Q_-^\dagger \frac{i\partial^+}{2} Q_- - \frac{i}{2}(\partial^+ Q_-^\dagger) Q_- - Q_+^\dagger \gamma_0 \gamma_i \frac{i\partial_i}{2} Q_- + \frac{i}{2}(\partial_i Q_-^\dagger)\gamma_0\gamma_i Q_+ \\
& -Q_-^\dagger \gamma_0 \gamma_i \frac{i\partial_i}{2} Q_+ + \frac{i}{2}(\partial_i Q_+^\dagger)\gamma_0\gamma_i Q_- - m_Q Q_+^\dagger \frac{\gamma^-}{2} Q_- - m_Q Q_-^\dagger \frac{\gamma^+}{2} Q_+ \\
& +2gq_+^\dagger q_+ A_+ + 2gq_-^\dagger q_- A_- - gq_+^\dagger \gamma_0 \gamma_i A^i q_- - gq_-^\dagger \gamma_0 \gamma_i A^i q_+ + q_+^\dagger \frac{i\partial^-}{2} q_+ - \frac{i}{2}(\partial^- q_+^\dagger) q_+ \\
& +q_-^\dagger \frac{i\partial^+}{2} q_- - \frac{i}{2}(\partial^+ q_-^\dagger) q_- - q_+^\dagger \gamma_0 \gamma_i \frac{i\partial_i}{2} q_- + \frac{i}{2}(\partial_i q_-^\dagger)\gamma_0\gamma_i q_+ \\
& -q_-^\dagger \gamma_0 \gamma_i \frac{i\partial_i}{2} q_+ + \frac{i}{2}(\partial_i q_+^\dagger)\gamma_0\gamma_i q_- \} \frac{dx^- dx^+ d^2x_\perp}{2}
\end{aligned} \tag{2.6}$$

where $Q_\pm = \Lambda_\pm Q$ and $q_\pm = \Lambda_\pm q$, $\Lambda_\pm = \frac{1}{2}\gamma^0\gamma^\pm$. Note also that $\partial_\nu = \frac{\partial}{\partial x^\nu}$ so that $\partial^- = 2\partial_+ = 2\frac{\partial}{\partial x^+}$ and $\partial^+ = 2\partial_- = 2\frac{\partial}{\partial x^-}$. The corresponding conjugate momenta for x^+-derivatives are

$$\pi^i_{Aa} = \frac{\partial \mathcal{L}}{\partial(\partial_+ A_{ia})} = -F_a^{+i} \tag{2.7}$$

$$\pi_Q(x) = \frac{\partial \mathcal{L}}{\partial(\partial_+ Q)} = iQ_+^\dagger \tag{2.8}$$

$$\pi_{Q^\dagger}(x) = \frac{\partial \mathcal{L}}{\partial(\partial_+ Q^\dagger)} = -iQ_+ \tag{2.9}$$

$$\pi_q(x) = \frac{\partial \mathcal{L}}{\partial(\partial_+ q)} = iq_+^\dagger \tag{2.10}$$

$$\pi_{q^\dagger}(x) = \frac{\partial \mathcal{L}}{\partial(\partial_+ q^\dagger)} = -iq_+ \tag{2.11}$$

For the momenta corresponding to x^--derivatives we get similar forms:

$$\rho^i_{Aa} = \frac{\partial \mathcal{L}}{\partial(\partial_- A_{ia})} = -F_a^{-i} \tag{2.12}$$

$$\rho_Q(x) = \frac{\partial \mathcal{L}}{\partial(\partial_- Q)} = iQ_-^\dagger \tag{2.13}$$

$$\rho_{Q^\dagger}(x) = \frac{\partial \mathcal{L}}{\partial(\partial_- Q^\dagger)} = -iQ_- \tag{2.14}$$

$$\rho_q(x) = \frac{\partial \mathcal{L}}{\partial(\partial_- q)} = iq_-^\dagger \tag{2.15}$$

$$\rho_{q^\dagger}(x) = \frac{\partial \mathcal{L}}{\partial(\partial_- q^\dagger)} = -iq_- \tag{2.16}$$

We rewrite $\mathcal{L} d^4x$ in the following way

$$\mathcal{L} d^4x = \frac{1}{2}\{\pi^i_{Aa}\partial_+ A_{ia} + \pi^i_{Aa}\partial_+ A_{ia} + \rho^i_{Aa}\partial_- A_{ia} + \rho^i_{Aa}\partial_- A_{ia}$$

$$+\pi_Q\partial_+Q_+ + \pi_Q\partial_+Q_+ + (\partial_+Q_+^\dagger)\pi_{Q^\dagger} + (\partial_+Q_+^\dagger)\pi_{Q^\dagger}$$

$$+\pi_q\partial_+q_+ + \pi_q\partial_+q_+ + (\partial_+q_+^\dagger)\pi_{q^\dagger} + (\partial_+q_+^\dagger)\pi_{q^\dagger}$$

$$+\rho_Q\partial_-Q_- + \rho_Q\partial_-Q_- + (\partial_-Q_-^\dagger)\rho_{Q^\dagger} + (\partial_-Q_-^\dagger)\rho_{Q^\dagger}$$

$$+\rho_q\partial_-q_- + \rho_q\partial_-q_- + (\partial_-q_-^\dagger)\rho_{q^\dagger} + (\partial_-q_-^\dagger)\rho_{q^\dagger}\}d^4x$$

$$-\mathcal{H}dx^4 - \mathcal{K}dx^4 + \mathcal{M}d^4x \tag{2.17}$$

The meaning of these terms is as in the previous papers [3], [1], so we don't go in detail here. The Hamiltonian densities $\mathcal{H}$ and $\mathcal{K}$ are :

$$\mathcal{H} = \{\frac{1}{2}(B_a^{-2}+E_a^{-2})-2gQ_+^\dagger Q_+A_+-2gQ_-^\dagger Q_-A_-+Q_+^\dagger\gamma_0\gamma_i\frac{i\partial_i}{2}Q_-+gQ_+^\dagger\gamma_0\gamma_i\frac{A^i}{2}Q_-$$

$$-\frac{i}{2}(\partial_iQ_-^\dagger)\gamma_0\gamma_iQ_++gQ_-^\dagger\gamma_0\gamma_i\frac{A^i}{2}Q_++m_QQ_+^\dagger\frac{\gamma^-}{4}Q_-+m_QQ_-^\dagger\frac{\gamma^+}{4}Q_+-2gq_+^\dagger q_+A_+-2gq_-^\dagger q_-A_-$$

$$+q_+^\dagger\gamma_0\gamma_i\frac{i\partial_i}{2}q_- + gq_+^\dagger\gamma_0\gamma_i\frac{A^i}{2}q_- - \frac{i}{2}(\partial_iq_-^\dagger)\gamma_0\gamma_iq_+ + gq_-^\dagger\gamma_0\gamma_i\frac{A^i}{2}q_+\} \tag{2.18}$$

$$\mathcal{K} = \{\frac{1}{2}(B_a^{+2}+E_a^{+2})-2gQ_-^\dagger Q_-A_--2gQ_+^\dagger Q_+A_++Q_-^\dagger\gamma_0\gamma_i\frac{i\partial_i}{2}Q_++gQ_-^\dagger\gamma_0\gamma_i\frac{A^i}{2}Q_+$$

$$-\frac{i}{2}(\partial_iQ_+^\dagger)\gamma_0\gamma_iQ_-+gQ_+^\dagger\gamma_0\gamma_i\frac{A^i}{2}Q_-+m_QQ_-^\dagger\frac{\gamma^-}{4}Q_++m_QQ_+^\dagger\frac{\gamma^+}{4}Q_--2gq_-^\dagger q_-A_--2gq_+^\dagger q_+A_+$$

$$+q_-^\dagger\gamma_0\gamma_i\frac{i\partial_i}{2}q_+ + gq_-^\dagger\gamma_0\gamma_i\frac{A^i}{2}q_+ - \frac{i}{2}(\partial_iq_-^\dagger)\gamma_0\gamma_iq_+ + gq_+^\dagger\gamma_0\gamma_i\frac{A^i}{2}q_-\} \tag{2.19}$$

where $B_a^- = B_a^+ = F_a^{12}$,$E_a^+ = -E_a^- = -\frac{1}{2}F_a^{+-}$ and for the "constraints" in the weak sense of Faddeev and Jackiw fj] we get whatever is left over:

$$\mathcal{M} = \{ -\partial_i A_{+a} F^{i+a} - \partial_i A_{-a} F^{i-a} - g A_{+a} A_{ib} f^{abc} F_c^{i+} - g A_{-a} A_{ib} f^{abc} F_c^{i-}$$

$$+F_{+-a}F^{+-a} - 2gA_{+a}Q_+^\dagger T^a Q_+ - 2gA_{-a}Q_-^\dagger T^a Q_- - 2gA_{+a}q_+^\dagger T^a q_+ - 2gA_{-a}q_-^\dagger T^a q_- \} \tag{2.20}$$

Well, we can rewrite is as (up to total derivatives) :

$$\mathcal{M} = A_{+a} C_\pi^a + A_{-a} C_\rho^a \tag{2.21}$$

and the "constraints" C_π^a and C_ρ^a are

$$C_\pi^a = -\partial_- F^{-+a} - \partial_i F^{i+a} - g f^{abc} A_{ib} F_c^{i+} - 2gQ_+^\dagger T^a Q_+ - 2gq_+^\dagger T^a q_+ \tag{2.22}$$

$$C_\rho^a = -\partial_+ F^{+-a} - \partial_i F^{i-a} - g f^{abc} A_{ib} F_c^{i-} - 2gQ_-^\dagger T^a Q_- - 2gq_-^\dagger T^a q_- \tag{2.23}$$

We want now $\mathcal{M} = 0$ so that we can identify unambiguously $\mathcal{H}$ and $\mathcal{K}$. We'll explore how to do this in the next section.

Let us write the $\mathcal{L}dx^4$ with the explicit momenta dependence (up to total derivatives which we can discard [8], [9] even for fermions [11]) so as to make the resulting commutation relation clear :

$$\mathcal{L}d^4x = \frac{1}{2}\{\pi^i_{Aa} dA_i^a - A_{ia} d\pi_A^{ia} + \pi_Q dQ_+ - d\pi_Q Q_+ + dQ_+^\dagger \pi_{Q^\dagger} - Q_+^\dagger d\pi_{Q^\dagger}$$

$$+\pi_q dq_+ - d\pi_q q_+ + dq_+^\dagger \pi_{q^\dagger} - q_+^\dagger d\pi_{q^\dagger}\}\frac{dx^- dx_\perp}{2}$$

$$+\frac{1}{2}\{\rho^i_{Aa} dA_i^a - A_{ia} d\rho_A^{ia} + \rho_Q dQ_+ - d\rho_Q Q_+ + dQ_+^\dagger \rho_{Q^\dagger} - Q_+^\dagger d\rho_{Q^\dagger}$$

$$+\rho_q dq_+ - d\rho_q q_+ + dq_+^\dagger \rho_{q^\dagger} - q_+^\dagger d\rho_{q^\dagger}\}\frac{dx^+ dx_\perp}{2}$$

$$-\mathcal{H}dx^4 - \mathcal{K}dx^4 \tag{2.24}$$

We see now that we have two types of evolutions, one along x^+, and one along x^-. According to Faddeev and Jackiw [8], we can read off the coefficients of the commutators, in their notation the two-form f_{ij}, in our notation the two-forms Γ_{ij} **and** Δ_{ij}, from the first term of (2.24). The commutation relations along surfaces $x^+ = y^+$ are then (the index i is base twelve):

$$[\xi^i, \xi^i] = \Gamma_{ij}^{-1} \quad i, j = 1, ..C \tag{2.25}$$

with

$$\xi^1 = \pi_A^1, \xi^2 = \pi_A^2, \xi^3 = \pi_Q, \xi^4 = \pi_{Q^\dagger}, \xi^5 = \pi_q, \xi^6 = \pi_{q^\dagger}$$

$$\xi^7 = A^1, \xi^8 = A^2, \xi^9 = Q_+, \xi^A = Q_+^\dagger \xi^B = q_+, \xi^C = q_+^\dagger \tag{2.26}$$

and

$$\Gamma_{17} = \Gamma_{28} = \Gamma_{39} = \Gamma_{4A} = \Gamma_{5B} = \Gamma_{6C} = 1 \tag{2.27}$$

$$\Gamma_{71} = \Gamma_{82} = \Gamma_{93} = \Gamma_{A4} = \Gamma_{B5} = \Gamma_{C5} = -1 \tag{2.28}$$

and all the other Γ's 0 .

The second term in equation (2.24) gives the commutation relations along surfaces $x^- = y^-$ according to the form :

$$[\eta^i, \eta^j] = \Delta^{-1}_{ij} \quad i, j = 1, ..C \tag{2.29}$$

with

$$\eta^1 = \rho^1_A, \eta^2 = \rho^2_A, \eta^3 = \rho_Q, \eta^4 = \rho_{Q^\dagger}, \eta^5 = \rho_q, \eta^6 = \rho_{q^\dagger}$$

$$\eta^7 = A^1, \eta^8 = A^2, \eta^9 = Q_-, \xi^A = Q^\dagger_- \eta^B = q_-, \eta^C = q^\dagger_- \tag{2.30}$$

and

$$\Delta_{17} = \Delta_{28} = \Delta_{39} = \Delta_{4A} = \Delta_{5B} = \Delta_{6C} = 1 \tag{2.31}$$

$$\Delta_{71} = \Delta_{82} = \Delta_{93} = \Delta_{A4} = \Delta_{B5} = \Delta_{C5} = -1 \tag{2.32}$$

and all the other Δ's 0. This form Δ is like the form Γ, a two-form, in our case invertible, which tells us that we don't require any further equations to identify the independent degrees of freedom [8], nor do we need to follow Dirac's method of quantization [12] [13] .

Going now to the quantum commutators, we get the following relations for fields at equal $x^+ = y^+$, the usual front-form "time" :

$$[A^i_{Pa}(x^+, x^-, x_\perp), \pi^j_{Ab}(y^+, y^-, y_\perp)]_{x^+=y^+} = i\delta(x^- - y^-)\delta^2(x_\perp - y_\perp)\delta^{ij}\delta_{ab} \tag{2.33}$$

$$\left\{Q_+(x^+, x^-, x_\perp), \pi_Q(y^+, y^-, y_\perp)\right\}_{x^+=y^+} = +i\Lambda_+\delta(x^- - y^-)\delta^2(x_\perp - y_\perp) \tag{2.34}$$

$$\left\{Q^\dagger_+(x^+, x^-, x_\perp), \pi_{Q^\dagger}(y^+, y^-, y_\perp)\right\}_{x^+=y^+} = -i\Lambda_+\delta(x^- - y^-)\delta^2(x_\perp - y_\perp) \tag{2.35}$$

$$\left\{q_+(x^+,x^-,x_\perp),\pi_q(y^+,y^-,y_\perp)\right\}_{x^+=y^+} = +i\Lambda_+\delta(x^- - y^-)\delta^2(x_\perp - y_\perp) \quad (2.36)$$

$$\left\{q_+^\dagger(x^+,x^-,x_\perp),\pi_{q^\dagger}(y^+,y^-,y_\perp)\right\}_{x^+=y^+} = -i\Lambda_+\delta(x^- - y^-)\delta^2(x_\perp - y_\perp) \quad (2.37)$$

Thus, the physical (quantized) degrees of freedom on $x^+ = 0$ are A^i_{Pa} , Q_+ ,$Q^\dagger_+$, q_+ and $q^\dagger_+$.

For fields at equal $x^- = y^-$, a new front-form "time", we get:

$$[A^i_{Ma}(x^+,x^-,x_\perp),\rho^j_{Ab}(y^+,y^-,y_\perp)]_{x^-=y^-} = i\delta(x^+ - y^+)\delta^2(x_\perp - y_\perp)\delta^{ij}\delta_{ab} \quad (2.38)$$

$$\left\{Q_-(x^+,x^-,x_\perp),\rho_Q(y^+,y^-,y_\perp)\right\}_{x^-=y^-} = +i\Lambda_-\delta(x^+ - y^+)\delta^2(x_\perp - y_\perp) \quad (2.39)$$

$$\left\{Q_-^\dagger(x^+,x^-,x_\perp),\rho_{Q^\dagger}(y^+,y^-,y_\perp)\right\}_{x^-=y^-} = -i\Lambda_-\delta(x^+ - y^+)\delta^2(x_\perp - y_\perp) \quad (2.40)$$

$$\left\{q_-(x^+,x^-,x_\perp),\rho_q(y^+,y^-,y_\perp)\right\}_{x^-=y^-} = +i\Lambda_-\delta(x^+ - y^+)\delta^2(x_\perp - y_\perp) \quad (2.41)$$

$$\left\{q_-^\dagger(x^+,x^-,x_\perp),\rho_{q^\dagger}(y^+,y^-,y_\perp)\right\}_{x^-=y^-} = -i\Lambda_-\delta(x^+ - y^+)\delta^2(x_\perp - y_\perp) \quad (2.42)$$

Here, the physical (quantized) degrees of freedom on $x^- = 0$ are A^i_{Ma} , Q_- ,$Q^\dagger_-$, q_- and $q^\dagger_-$. Note that A^+_a and A^-_a do not enter in the list of physical (quantized) degrees of freedom.

The equations of motions are now like in Faddeev and Jackiw [8]

$$\Gamma_{ij}\partial_+\xi^j = \frac{\partial\mathcal{H}}{\partial\xi^i} \quad (2.43)$$

for the x^+ variation, and

$$\Delta_{ij}\partial_-\eta^j = \frac{\partial\mathcal{K}}{\partial\eta^i} \quad (2.44)$$

for the x^- variation . For $i = 7$ and $j = 1, 2$, equation (2.43) gives

$$-\partial_+ F^{+ia} = +2gQ_+^\dagger \gamma_0 \gamma_i T^a Q_- + 2gQ_-^\dagger \gamma_0 \gamma_i T^a Q_+ + 2gq_+^\dagger \gamma_0 \gamma_i T^a q_- + 2gq_-^\dagger \gamma_0 \gamma_i T^a q_+$$

$$+\partial_- F^{-ia} + \partial_j F^{jia} + gf^{abc} A_{-b} F_c^{-i} + gf^{abc} A_{jb} F_c^{ji} \tag{2.45}$$

which is just the equation of motion [(2.3)] for $\nu = i$. For $i = 9$ and $j = 3$ we recover the equation of motion for $Q_+^\dagger$

$$i\partial^- Q_+^\dagger = i\partial_i Q_-^\dagger \gamma_0 \gamma_i + gQ_-^\dagger \gamma_0 \gamma_i A^i - m_Q Q_-^\dagger \gamma_0 - 2gQ_+^\dagger A_+ \tag{2.46}$$

We get similar results from (2.44):

$$i\partial^+ Q_-^\dagger = i\partial_i Q_+^\dagger \gamma_0 \gamma_i + gQ_+^\dagger \gamma_0 \gamma_i A^i - m_Q Q_+^\dagger \gamma_0 - 2gQ_-^\dagger A_- \tag{2.47}$$

3. Quantization of the Fields

The quantization of the fields $A^i_{P,Mb}$ $Q_\pm$ and $q_\pm$ goes through as in the Abelian case [1], [3]. It is only in the components A_a^+ and A_a^- that any surprises might arise. For completeness, we write out the resulting fields anyway.

On $x^+ = 0$ we have the following definitions for the "plus-moving" fields:

$$A_{iPb}(x^-, x_\perp) = \int \frac{d^2 k_\perp}{(2\pi)^3} \frac{dk^+}{2k^+} \sum_\lambda \left\{ \epsilon_i^\lambda a_b(k^+, k_\perp, \lambda) e^{-ik.x} + \epsilon_i^{*\lambda} a_b^\dagger(k^+, k_\perp, \lambda) e^{+ik.x} \right\} \tag{3.1}$$

$$Q_+(x^-, x_\perp) = \int \frac{d^2 k_\perp}{(2\pi)^3} \frac{dk^+}{2k^+} \sum_\lambda \left\{ b(k^+, k_\perp) u_{Q+}(k^+, k_\perp, \lambda) e^{-ik.x} \right.$$

$$+d^\dagger(k^+,k_\perp)v_{Q+}(k^+,k_\perp,\lambda)e^{+ik.x}\} \tag{3.2}$$

$$q_+(x^-,x_\perp) = \int \frac{d^2k_\perp}{(2\pi)^3}\frac{dk^+}{2k^+}\sum_\lambda \{b(k^+,k_\perp)u_{q+}(k^+,k_\perp,\lambda)e^{-ik.x}$$

$$+d^\dagger(k^+,k_\perp)v_{q+}(k^+,k_\perp,\lambda)e^{+ik.x}\} \tag{3.3}$$

As before, $ik.x = \frac{i}{2}k^+x^- - ik_\perp.x_\perp$ and the polarization vectors are $\epsilon_i^{+1} = 1/\sqrt{2}(1,i);\, \epsilon_i^{-1} = 1/\sqrt{2}(1,-i)$.

On the other null-surface $x^- = 0$ we get similar forms for the "minus-movers:"

$$A_{iMb}(x^+,x_\perp) = \int \frac{d^2k_\perp}{(2\pi)^3}\frac{dk^-}{2k^-}\sum_\mu \{\epsilon_i^\mu \hat{a}_b(k^-,k_\perp,\mu)e^{-i\hat{k}.x} + \epsilon_i^{*\mu}\hat{a}_b^\dagger(k^-,k_\perp,\mu)e^{+i\hat{k}.x}\} \tag{3.4}$$

$$Q_-(x^+,x_\perp) = \int \frac{d^2k_\perp}{(2\pi)^3}\frac{dk^-}{2k^-}\sum_\mu \{\hat{b}(k^-,k_\perp)u_{Q-}(k^-,k_\perp,\mu)e^{-i\hat{k}.x}$$

$$+\hat{d}^\dagger(k^-,k_\perp)v_{Q-}(k^-,k_\perp,\mu)e^{+i\hat{k}.x}\} \tag{3.5}$$

$$q_-(x^+,x_\perp) = \int \frac{d^2k_\perp}{(2\pi)^3}\frac{dk^-}{2k^-}\sum_\mu \{\hat{b}(k^-,k_\perp)u_{q-}(k^-,k_\perp,\mu)e^{-i\hat{k}.x}$$

$$+\hat{d}^\dagger(k^-,k_\perp)v_{q-}(k^-,k_\perp,\mu)e^{+i\hat{k}.x}\} \tag{3.6}$$

Here , $i\hat{k}.x = \frac{i}{2}k^-x^+ - ik_\perp.x_\perp$ and the polarization vector is like before.

On $x^+ = 0$, $A_{iPb}(x^-,x_\perp)$ creates or destroys vector quanta with momentum $k = (k^+,k_\perp)$, polarization $\epsilon_i(k^+,k_\perp)$ and color label b. On $x^- = 0$, $A_{iMb}(x^+,x_\perp)$ creates or destroys quanta with momentum $\hat{k} = (k^-,k_\perp)$,polarization $\epsilon_i(k^-,k_\perp)$ and color label b.

The analysis for the fermion fields goes through just like in the previous papers [3], [1], [4].

As before , the fields are defined thus on the initializing surfaces $x^+ = 0$, $x^- = 0$, and $x^+ = x^- = 0$

$$A_i(x^+, x^-, x_\perp) = A_{iP}(x^-, x_\perp) + A_{iM}(x^+, x_\perp) - B_i(x_\perp) \tag{3.7}$$

where the last term insures that the field A^i is defined consistently at $x^+ = x^- = 0$:

$$B_i(x_\perp) = A_{iP}(0, x_\perp) = A_{iM}(0, x_\perp) \tag{3.8}$$

The fermion fields are defined to be

$$Q(x^+, x^-, x_\perp) = Q_-(x^+, x_\perp) + Q_+(x^-, x_\perp) \tag{3.9}$$

and

$$q(x^+, x^-, x_\perp) = q_-(x^-, x_\perp) + q_+(x^+, x_\perp) \tag{3.10}$$

We also have a continuity relation between the $_+$ and $_-$ projections of the fermionic fields:

$$Q_-(0, x_\perp) = D_1 Q_+(0, x_\perp)$$

$$q_-(0, x_\perp) = D_2 q_+(0, x_\perp) \tag{3.11}$$

for some matrices D_i.

Since the fields A^+, A^- go into defining the operators $\mathcal{H}$ and $\mathcal{K}$, we need to know their behaviour on $x^+ = 0$ and $x^- = 0$. Furthermore, to identify uniquely $\mathcal{H}$ and $\mathcal{K}$, we need $\mathcal{M}$ to be zero on the initializing surfaces.

On the surface $x^+ = 0$, our solution to $\mathcal{M} = 0$ is to set $C_\pi = A^+_P = 0$ [14] . This is (with some rearrangement)

$$\partial^+(-\frac{1}{2}\partial^+ A^-_{Pa} - \partial_i A^i_{Pa}) = -2gQ^\dagger_+ T_a Q_+ - 2gq^\dagger_+ T_a q_+ + gf_{abc}A^b_{Pi}\partial^+ A^{ic}_P \quad (3.12)$$

Since on this surface the fields depend on $(x^-, x_\perp)$, we get a great simplification. As a matter of fact , we can easily solve this equation and obtain

$$A^-_{Pa} = -\frac{2}{(i\partial^+)} i\partial_i A^i_{Pa} - \frac{2}{(i\partial^+)^2} igf_{abc}A^b_{Pi} i\partial^+ A^{ic}_P - \frac{4}{(i\partial^+)^2} gQ^\dagger_+ T_a Q_+ - \frac{4}{(i\partial^+)^2} gq^\dagger_+ T_a q_+ \quad (3.13)$$

To fully define these fields, we need to define the operators $((i\partial^+)^{-1}$ and $((i\partial^+)^2)^{-1}$. We use the definitions of the previous papers as well [3], [1], as well as the definitions of Zhang and Harindrananth [9], who use anti-periodic boundary conditions to define these operators, which we do as well.

For the fields on $x^- = 0$, we get similar results, noting though that the fields depend only on $(x^+, x_\perp)$. We get $\mathcal{M} = 0$ by setting $A^-_M = 0 = C_\rho$ and obtain

$$A^+_{Ma} = -\frac{2}{(i\partial^-)} i\partial_i A^i_{Ma} - \frac{2}{(i\partial^-)^2} igf_{abc}A^b_{Mi} i\partial^- A^{ic}_M - \frac{4}{(i\partial^-)^2} gQ^\dagger_- T_a Q_- - \frac{4}{(i\partial^-)^2} gq^\dagger_- T_a q_- \quad (3.14)$$

Again, to fully define this operator, we need to specify what do we mean by $(i\partial^-)^{-1}$ and $(i\partial^-)^{-2}$ We also follow the approach of the previous work [3], [1].

The question might arise as to whether this choice of $\mathcal{M} = 0$ is not a gauge choice, like the usual light-cone gauge choice $A^+ = 0$. It seems that in some sense it is, but in a particular fashion of this null-surfaces quantization, it is realized on **both** Σ_+ and Σ_-! Thus, we have both $A^+_P = 0$ and $A^-_M = 0$. Yet it is not a gauge choice in the usual sense, because $A^+ \neq 0$ and neither is A^- zero! Implicitly,

$A^+ = A^+_P + A^+_M$ and $A^- = A^-_P + A^-_M$. In particular, A^-_P and A^+_M are given above, neither of which is zero, hence neither A^+ not A^- is zero. Since these definitions are on two "halves" of the old $t = 0$ line, together they make up the usual light-cone A^-.

At the same time, the conditions $C_\pi = 0$ and $C_\rho = 0$ are like Gauss's law, but here imposed on $\Sigma_\pm$.

4. Perturbation Theory Solution for Heavy Quark Limit of QCD

Let us study now the Hamiltonian P^-. It is constructed out of the Hamiltonian density $\mathcal{H}$. But first, we must express all the fields in terms of the fields defined on $x^+ = 0$. We have

$$\mathcal{H} = \{\frac{1}{2}(B^{-2}_{Pa} + E^{-2}_{Pa}) - gQ^\dagger_+ Q_+ A^-_P + Q^\dagger_+ \gamma_0 \gamma_i \frac{i\partial_i}{2} Q_- + gQ^\dagger_+ \gamma_0 \gamma_i \frac{A^i_P}{2} Q_-$$

$$+Q^\dagger_- \gamma_0 \gamma_i \frac{i\partial_i}{2} Q_+ + gQ^\dagger_- \gamma_0 \gamma_i \frac{A^i_P}{2} Q_+ + m_Q Q^\dagger_+ \frac{\gamma^-}{4} Q_- + m_Q Q^\dagger_- \frac{\gamma^+}{4} Q_+$$

$$-gq^\dagger_+ q_+ A^-_P + q^\dagger_+ \gamma_0 \gamma_i \frac{i\partial_i}{2} q_- + gq^\dagger_+ \gamma_0 \gamma_i \frac{A^i_P}{2} q_-$$

$$+q^\dagger_- \gamma_0 \gamma_i \frac{i\partial_i}{2} q_+ + qQ^\dagger_- \gamma_0 \gamma_i \frac{A^i_P}{2} q_+\} \tag{4.1}$$

where

$$B^-_{Pa} = (\partial^1 A^2_{Pa} - \partial^2 A^1_{Pa}), E^-_{Pa} = -\frac{1}{2}\partial^+ A^-_{Pa} \tag{4.2}$$

These terms take these particularly simple forms due to our choice of partial "gauge" $A^+_P = 0$.

We follow the notation of [9] and observe that the fermionic fields are

$$Q_{+}^{\dagger} = (\xi^{\dagger}, 0) \tag{4.3}$$

and

$$Q_{-}^{\dagger} = (0, \eta^{\dagger}) \tag{4.4}$$

so that we have

$$\eta = \frac{1}{i\partial^{+}} \{ i\partial_i \sigma_i + \sigma_i g A_P^i + i m_Q \} \xi \tag{4.5}$$

as well as

$$q_{+}^{\dagger} = (\sigma^{\dagger}, 0) \tag{4.6}$$

and

$$q_{-}^{\dagger} = (0, \tau^{\dagger}) \tag{4.7}$$

so that we have

$$\sigma = \frac{1}{i\partial^{+}} \{ i\partial_i \sigma_i + \sigma_i g A_P^i \} \tau \tag{4.8}$$

where σ_i are the 2x2 Pauli sigma matrices and now the spinors ξ, η, σ and τ are two-component spinors [9].

Substituting this in the equation [(4.1)] we get

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I \tag{4.9}$$

where

$$\mathcal{H}_0 = \frac{1}{2} A_{Pa}^i T^a \partial_i^2 A_{Pb}^i T^b + \frac{1}{2} \xi^{\dagger} \frac{(-\partial_i^2 + m_Q^2)}{i\partial^{+}} \xi + \frac{1}{2} \sigma^{\dagger} \frac{(-\partial_i^2)}{i\partial^{+}} \sigma \tag{4.10}$$

while the interaction Hamiltonian density is much more complicated [15] . In the non-Abelian theory, the presence of non-linear gauge coupling gives rise to more complicated terms:

$$H_I = \mathcal{H}_{AAA} + \mathcal{H}_{Q\overline{Q}A} + \mathcal{H}_{AAAA} + \mathcal{H}_{Q\overline{Q}AA} + \mathcal{H}_{Q\overline{Q}Q\overline{Q}} + \mathcal{H}_{Q\overline{Q}q\overline{q}}$$

$$+\mathcal{H}_{q\overline{q}A} + H_{q\overline{q}AA} + \mathcal{H}_{q\overline{q}q\overline{q}} + \mathcal{H}_{q\overline{q}Q\overline{Q}} \tag{4.11}$$

with

$$\mathcal{H}_{AAA} = -igf^{abc}A^i_{Pa}A^j_{Pb}i\partial^i A^j_{Pc} - igf_{abc}i\partial_i A^i_{Pa}\frac{1}{i\partial^+}A^b_{Pi}i\partial^+ A^{ic}_P \tag{4.12}$$

$$\mathcal{H}_{AAAA} = \frac{g^2}{4}f^{abc}f^{ade}A^i_{Pb}A^j_{Pc}A^i_{Pd}A^j_{Pe} + \frac{g^2}{2}f^{abc}f^{ade}\frac{1}{i\partial^+}A^i_{Pb}i\partial^+ A^i_{Pc}\frac{1}{i\partial^+}A^j_{Pd}i\partial^+ A^j_{Pe} \tag{4.13}$$

$$\mathcal{H}_{Q\overline{Q}A} = +g\xi^\dagger\frac{i\partial_k}{i\partial^+}A^k_{Pa}T^a\xi - \frac{g}{2}\xi^\dagger\sigma_i A^i_{Pa}T^a\frac{1}{i\partial^+}(\sigma_j i\partial_j + im)\xi - \frac{g}{2}\xi^\dagger\frac{1}{i\partial^+}(\sigma_j i\partial_j - im)\sigma_i A^i_{Pa}T^a\xi$$

$$+g\xi^\dagger i\partial_k A^k_{Pa}T^a\frac{1}{i\partial^+}\xi - \frac{g}{2}\xi^\dagger\sigma_i(\sigma_j i\partial_j + im)\frac{1}{i\partial^+}A^i_{Pa}T^a\xi - \frac{g}{2}\xi^\dagger\sigma_i A^i_{Pa}T^a(\sigma_j i\partial_j - im)\frac{1}{i\partial^+}\xi$$

$$+2g\xi^\dagger T^a\xi\frac{1}{i\partial^+}i\partial_i A^i_{Pa} \tag{4.14}$$

$$\mathcal{H}_{Q\overline{Q}AA} = +2ig^2 f_{abc}\xi^\dagger T^a\xi\frac{1}{i\partial^+}A^b_{Pi}A^{ci}_P - 2ig^2 f_{abc}\frac{1}{i\partial^+}\xi^\dagger T^a\xi\frac{1}{i\partial^+}A^b_{Pi}i\partial^+ A^{ci}_P$$

$$+g^2\xi^\dagger\sigma_i A^i_{Pa}T^a\frac{1}{i\partial^+}\sigma_j A^j_{Pb}T^b\xi \tag{4.15}$$

$$\mathcal{H}_{Q\overline{Q}Q\overline{Q}} = 4g^2\xi^\dagger T_a\xi\frac{1}{(i\partial^+)^2}\xi^\dagger T^a\xi - 2g^2\frac{1}{i\partial^+}\xi^\dagger T_a\xi\frac{1}{i\partial^+}\xi^\dagger T^a\xi \tag{4.16}$$

$$\mathcal{H}_{Q\overline{Q}q\overline{q}} = 4g^2\xi^\dagger T_a\xi\frac{1}{(i\partial^+)^2}\sigma^\dagger T^a\sigma - 2g^2\frac{1}{i\partial^+}\xi^\dagger T_a\xi\frac{1}{i\partial^+}\sigma^\dagger T^a\sigma \tag{4.17}$$

and

$$\mathcal{H}_{q\overline{q}A} = +g\sigma^\dagger \frac{i\partial_k}{i\partial^+} A^k_{Pa} T^a \sigma - \frac{g}{2}\sigma^\dagger \sigma_i A^i_{Pa} T^a \frac{1}{i\partial^+}\sigma_j i\partial_j \sigma - \frac{g}{2}\sigma^\dagger \frac{1}{i\partial^+}\sigma_j i\partial_j \sigma_i A^i_{Pa} T^a \sigma$$

$$+g\sigma^\dagger i\partial_k A^k_{Pa} T^a \frac{1}{i\partial^+}\sigma - \frac{g}{2}\sigma^\dagger \sigma_i \sigma_j i\partial_j \frac{1}{i\partial^+} A^i_{Pa} T^a \sigma - \frac{g}{2}\sigma^\dagger \sigma_i A^i_{Pa} T^a \sigma_j i\partial_j \frac{1}{i\partial^+}\sigma$$

$$+2g\sigma^\dagger T^a \sigma \frac{1}{i\partial^+} i\partial_i A^i_{Pa} \tag{4.18}$$

$$\mathcal{H}_{q\overline{q}AA} = +2ig^2 f_{abc}\sigma^\dagger T^a \sigma \frac{1}{i\partial^+} A^b_{Pi} A^{ci}_P - 2ig^2 f_{abc} \frac{1}{i\partial^+}\sigma^\dagger T^a \sigma \frac{1}{i\partial^+} A^b_{Pi} (i\partial^+) A^{ci}_P$$

$$+g^2 \sigma^\dagger \sigma_i A^i_{Pa} T^a \frac{1}{i\partial^+}\sigma_j A^j_{Pb} T^b \sigma \tag{4.19}$$

$$\mathcal{H}_{q\overline{q}q\overline{q}} = 4g^2 \sigma^\dagger \sigma \frac{1}{(i\partial^+)^2}\sigma^\dagger T_a T^a \sigma - 2g^2 \frac{1}{i\partial^+}\sigma^\dagger T_a \sigma \frac{1}{i\partial^+}\sigma^\dagger T^a \sigma \tag{4.20}$$

$$\mathcal{H}_{q\overline{q}Q\overline{Q}} = 4g^2 \sigma^\dagger \sigma \frac{1}{(i\partial^+)^2}\xi^\dagger T_a T^a \xi - 2g^2 \frac{1}{i\partial^+}\sigma^\dagger T_a \sigma \frac{1}{i\partial^+}\xi^\dagger T^a \xi \tag{4.21}$$

For $\mathcal{K}$ we get the similar results, with $\xi \rightarrow \eta$, $\sigma \rightarrow \tau$ and $P \rightarrow M$ in the above expression. Note that we get results similar to Harindranath and Zhang, [9], with the exception of several operators: $\mathcal{H}_{Q\overline{Q}A}$, $\mathcal{H}_{q\overline{q}A}$, $\mathcal{H}_{Q\overline{Q}AA}$, $\mathcal{H}_{q\overline{q}AA}$. For details, see Appendix B as well as Table I of Zhang and Harindranath [9].

5. One-Loop Fermion Wave-Function Renormalizations

We are ready to apply this to perturbation theory: we can calculate the fermion wave-function renormalizations to one-loop. Since there are two types of quarks, one heavy one light, we get two renormalization constants, Z_Q and Z_q. Let us compute the heavy quark wave-function renormalization Z_Q first. For this, we rely the perturbation theory on null-surfaces developed in our previous work on Abelian gauge theory [3], which in turn was a modification of the work of [9] and of Bjorken and collaborators [16] . See also Appendix B for details for the interaction terms which build up P_I^-.

Since both P^+ and P^- are dynamical, we get contributions from both of them. First, let us compute the contributions from P^-, denoted by δP^- (see details in [3] for the definition and use of δP^-), for a heavy quark with momentum p :

$$\delta P^- = \delta P_1^- + \delta P_2^- + \delta P_3^- \tag{5.1}$$

where

$$\delta P_1^- = (g)^2(T^aT^a)\int \frac{d^2k_\perp}{16\pi^3}dk^+\frac{\theta(k^+)\theta(p^+-k^+)}{[k^+]}\Gamma^i_{Q0}(p,p-k,k)*\Gamma^i_{Q0}(p-k,p,k)$$

$$\frac{1}{p^- - k^- - (p-k)^-} \tag{5.2}$$

which is due to the emission and reabsorption of a virtual gluon.

The second piece comes from the four lepton piece of the Hamiltonian, $\mathcal{H}_{Q\overline{Q}Q\overline{Q}}$, where two of the quarks have been "tied" together

$$\delta P_2^- = 2g^2(T^aT^a)\int \frac{d^2k_\perp}{16\pi^3}dk^+\theta(k^+)\{\frac{1}{[p^+-k^+]^2}-\frac{1}{[p^++k^+]^2}\} \tag{5.3}$$

The last piece comes from the two-gluon piece of the Hamiltonian, $\mathcal{H}_{Q\overline{Q}AA}$, where the gluon lines have been "tied" together

$$\delta P_3^- = g^2(T^aT^a)\int \frac{d^2k_\perp}{16\pi^3}\frac{dk^+\theta(k^+)}{[k^+]}\frac{\sigma_i\sigma^i}{[p^+-k^+]} \tag{5.4}$$

The first contribution becomes, after introducing the parameters $[x]=\frac{[k^+]}{p^+}$ and $\kappa_\perp = k_\perp - xp_\perp$:

$$\delta P_1^- = \frac{g^2}{p^+}C_F\int\frac{d(\kappa^2)}{16\pi^2}\int_0^1 dx\frac{1}{[1-x]}\frac{\kappa^2(\frac{4}{[x]^2}-\frac{4}{[x]}+2)}{x(1-x)p^2-\kappa^2-xm_Q^2} \tag{5.5}$$

We perform now transverse dimensional regularization, and obtain

$$\delta P_1^- = -\frac{g^2}{16\pi^2}C_F\{\frac{p^2-m_Q^2}{p^+}[\frac{1}{\epsilon_t}(2ln\frac{p^+}{\lambda}-\frac{3}{2})-\int_0^1 dx(\frac{2}{[x]}-2+x)ln(x\frac{m_Q^2}{\mu^2}-x(1-x)\frac{p^2}{\mu^2})]$$

$$-2\frac{p^2}{p^+}[\frac{3}{2\epsilon_t}-\int_0^1 dx(2-x)ln(x\frac{m_Q^2}{\mu^2}-x(1-x)\frac{p^2}{\mu^2})]\} \tag{5.6}$$

where $\epsilon_t = 1-\frac{d_t}{2}$ (so that $\frac{1}{\epsilon_t}$ is equivalent to $ln\frac{\Lambda_\perp^2}{m^2}$ in ultraviolet cut-off regularization), λ is the infra-red regularization and μ is a mass scale introduced by dimensional regularization [9]. The QCD factor C_F is defined to be $(T^aT^a)=C_F=\frac{N_c^2-1}{N_c}$ and N_c are the number of colors, $N_c=3$ in the usual case.

The second contribution, δP_2^- becomes

$$\delta P_2^- = \frac{2g^2}{p^+}C_F\int\frac{d^2k_\perp}{8\pi^2}\int_0^\infty dx(\frac{1}{[1-x]^2}-\frac{1}{(1+x)^2}) \tag{5.7}$$

Using transverse dimensional regularization, this is just zero:

$$\delta P_2^- = 0 \tag{5.8}$$

The last contribution, δP_3^- becomes

$$\delta P_3^- = \frac{g^2}{p^+} C_F \int \frac{d^2 k_\perp}{8\pi^2} \int_0^\infty \frac{dx}{[x]} \frac{2}{[1-x]} \tag{5.9}$$

Using transverse dimensional regularization, this is also zero.

The total contribution to δP^- is

$$\delta P^- = -\frac{g^2}{16\pi^2} C_F \{ \frac{p^2 - m_Q^2}{p^+} [\frac{1}{\epsilon_t} (2ln\frac{p^+}{\lambda} - \frac{3}{2}) - \int_0^1 dx (\frac{2}{[x]} - 2 + x) ln (x\frac{m_Q^2}{\mu^2} - x(1-x)\frac{p^2}{\mu^2})]$$

$$-2\frac{p^2}{p^+}[\frac{3}{2\epsilon_t} - \int_0^1 dx (2-x) ln (x\frac{m_Q^2}{\mu^2} - x(1-x)\frac{p^2}{\mu^2})] \} \tag{5.10}$$

We compute now easily the contributions from δP^+, by using the explicit parity symmetry of our formalism:

$$\delta P^+ = -\frac{g^2}{16\pi^2} C_F \{ \frac{p^2 - m_Q^2}{p^-} [\frac{1}{\epsilon_t} (2ln\frac{p^-}{\lambda} - \frac{3}{2}) - \int_0^1 dx (\frac{2}{[x]} - 2 + x) ln (x\frac{m_Q^2}{\mu^2} - x(1-x)\frac{p^2}{\mu^2})]$$

$$-2\frac{p^2}{p^-}[\frac{3}{2\epsilon_t} - \int_0^1 dx (2-x) ln (x\frac{m_Q^2}{\mu^2} - x(1-x)\frac{p^2}{\mu^2})] \} \tag{5.11}$$

with λ and ϵ_t as above.

Putting all of this together, we get the following for the correction to the heavy quark mass :

$$\delta M_Q^2 = p^+ \delta P^-_{p^2 = p^+ p^- = m_Q^2} + p^- \delta P^+_{p^2 = p^+ p^- = m_Q^2}$$

$$= \frac{3g^2}{8\pi^2} C_F m_Q^2 \frac{1}{\epsilon_t} \tag{5.12}$$

which is the standard covariant result [17] .

We can also obtain now the heavy quark wave-function renormalization.It is

$$Z_Q^{-1} = 1 - (\frac{\partial \delta P^-}{\partial p^-})_{p^2=p^+p^-=m_Q^2} - (\frac{\partial \delta P^+}{\partial p^+})_{p^2=p^+p^-=m_Q^2}$$

$$Z_Q^{-1} = 1 - \frac{g^2}{4\pi^2} C_F \{ \frac{1}{\epsilon_t}(3 - ln\frac{m_Q^2}{\lambda^2}) - ln\frac{m_Q^2}{\lambda^2} ln\frac{\mu^2}{m_Q^2} \} \tag{5.13}$$

Again, this is very similar to the usual calculations , down to the singularity structure, but with different coefficients than usual.

Since we see that the limit $m_Q \to 0$ is smooth, we can just take the results above and take this limit to obtain the massless quark corrections. We obtain what we expect,

$$\delta M_q^2 = 0 \tag{5.14}$$

and

$$Z_q^{-1} = 1 - \frac{3g^2}{4\pi^2} C_F \frac{1}{\epsilon_t} \tag{5.15}$$

so that the massless quark mass does not get renormalized to one-loop.

6. One-Loop Gluon Wave-Function and Vertex Renormalization

We look now at the one-loop quantum correction to the gluon wave-function and to the vertex renormalization. We proceed like we did in the previous work [3], except that now we have more vertices. Again, the meaning of δP^- is discussed in greater detail in [3]. See also Appendix B for definitions of the interaction Hamiltonians P_I^- . Let us look at the gluon wave-function renormalization.

We get several contributions: first, from the three gluon vertex; next, from the quarks coupling to the gluon, $\mathcal{H}_{Q\overline{Q}A}$, and from the light quark coupling to the gluon, $\mathcal{H}_{q\overline{q}A}$;the third one from the two-gluon two-quark vertex $H_{Q\overline{Q}AA}$ and $H_{Q\overline{Q}AA}$; and finally, the four gluon vertex $\mathcal{H}_{AAAA}$ give two contributions.

$$\delta P^- = \delta P_1^- + \delta P_2^- + \delta P_3^- + \delta P_4^- + \delta P_5^- \tag{6.1}$$

For the first piece, we have as follows

$$\delta P_1^- \delta^{ab} = \frac{(-ig)^2}{2q^+} f^{acd} f^{bdc} \int \frac{d^2k_\perp}{16\pi^3} dk^+ \frac{\theta(k^+)}{[k^+]} \frac{\theta(q^+ - k^+)}{[q^+ - k^+]} \Gamma_{g0}^{jlm}(q, -k, k-q) *$$

$$\Gamma_{g0}^{iml}(-q, q-k, k) \frac{1}{q^- - k^- - (q-k)^-} \} \tag{6.2}$$

where $\Gamma_{g0}^{jlm}(p,k,q)$ is the three-gluon vertex of Harindranath and Zhang [9] as well as our term $\mathcal{H}_{AAA}$ as detailed in Appendix B.

This gives a result equal to that of Harindranath and Zhang [9]:

$$\delta P_1^- = 2\frac{g^2}{q^+} C_A \int \frac{d^2\kappa}{16\pi^2} \int_0^1 dx \frac{\kappa^2}{x(1-x)q^2 - \kappa^2} [1 + \frac{1}{[x]^2} + \frac{1}{[1-x]^2}] \tag{6.3}$$

The QCD constant C_A is defined to be $C_A = N_c$ for $SU(N_c)$. This simplifies to

$$\delta P_1^- = -\frac{g^2}{16\pi^2} C_A \frac{q^2}{q^+} \{ (2ln\frac{q^+}{\lambda} - \frac{11}{6})(\frac{1}{\epsilon_t} - ln\frac{\mu^2}{q^2}) - (ln\frac{q^+}{\lambda})^2 \} \tag{6.4}$$

where the parameter μ, ϵ_t and λ are as before.

The next contribution for a gluon of momentum q is as follows (the overall minus sign comes from the fermion loop) :

$$\delta P_2^- \delta^{ab} = (-1)\frac{(-2g)^2}{q^+} Tr(T^a T^b) \int \frac{d^2 k_\perp}{16\pi^3} dk^+ \theta(k^+)\theta(q^+ - k^+) Tr\{\chi^\dagger \chi \frac{q_i}{[q^+]}\frac{q^i}{[q^+]}\chi^\dagger \chi\}$$

$$\frac{1}{q^- - k^- - (q-k)^-} \tag{6.5}$$

which becomes, upon using $[x] = \frac{[k^+]}{q^+}$ and $\kappa_\perp = k_\perp - x q_\perp$

$$\delta P_2^- = 4\frac{g^2}{q^+} T_F \int \frac{d\kappa^2}{16\pi^2} \int_0^1 dx \frac{(1-x)x q_\perp^2}{x(1-x)q^2 - \kappa^2 - m_Q^2} \tag{6.6}$$

Here, $T_F = \frac{1}{2}$. Using again transverse dimensional regularization, we get

$$\delta P_2^- = \frac{g^2}{16\pi^2} T_F \frac{q^+ q^- - q^2}{[q^+]} \frac{4}{6}\frac{1}{\epsilon_t} \tag{6.7}$$

where the meaning of ϵ_t and λ is as before. Since there are N_{LF} light flavors of quarks and N_{HF} heavy flavors, we multiply this result by $N_F = N_{LF} + N_{HF}$, as we obtain this contribution from each of the N_F quarks present. Notice that as in the QED case [3], we differ from Harindranth and Zhang [9] in this term, which gives us the vanishing of the gluon mass but which gives Harindranath and Zhang the term proportional to m_Q^2, thereby spoiling the one-loop gauge boson mass correction.

For the third contribution, from the light quark, we get

$$\delta P_3^- \delta^{ab} = \frac{g^2}{[q^+]} Tr(T^a T^b) \int d^2 k_\perp \frac{dk^+}{16\pi^3} \theta(k^+) T(\sigma_i \sigma^i)[\frac{1}{[p^+ - k^+]} - \frac{1}{[p^+ + k^+]}] \tag{6.8}$$

This becomes

$$\delta P_3^- = \frac{g^2}{[q^+]} 2T_F \int \frac{d^2k_\perp}{16\pi^3} \int_0^\infty dx [\frac{1}{[1-x]} - \frac{1}{[1+x]}] \tag{6.9}$$

which is zero by transverse dimensional regularization. The heavy quark also contributes zero.

For the fourth contribution, we get

$$\delta P_4^- \delta^{ab}\delta_{ij} = \frac{1}{2}\frac{g^2}{[q^+]} f^{acd} f^{bcd} \int \frac{d^2k_\perp}{16\pi^3} dk^+ \frac{\theta(k^+)}{[k^+]} \delta_{il}\delta_{jl} [\frac{(q^+ + k^+)^2}{[q^+ - k^+]^2} + [\frac{(q^+ - k^+)^2}{[q^+ + k^+]^2}] \tag{6.10}$$

This becomes

$$\delta P_4^- = \frac{g^2}{2[q^+]} C_A \int \frac{d^2k_\perp}{16\pi^3} \int_0^\infty \frac{dx}{[x]} [\frac{(1+x)^2}{[1-x]^2} + \frac{(1-x)^2}{[1+x]^2}] \tag{6.11}$$

which is also zero by transverse dimensional regularization.

Finally, the last term is

$$\delta P_5^- \delta^{ab}\delta_{ij} = \frac{1}{2}\frac{g^2}{[q^+]} f^{acd} f^{bcd} \int \frac{d^2k_\perp}{16\pi^3} dk^+ \frac{\theta(k^+)}{[k^+]} 2[\delta_{ij}\delta_{ll} - \delta_{il}\delta_{jl}] \tag{6.12}$$

This becomes

$$\delta P_5^- = \frac{g^2}{[q^+]} C_A \int \frac{d^2k_\perp}{16\pi^3} \int_0^\infty \frac{dx}{[x]} \tag{6.13}$$

which is also zero by transverse dimensional regularization.

We need now to compute the P^+'s. Again, we use the explicit parity covariance and write simply

$$\delta P_1^+ = -\frac{g^2}{16\pi^2} C_A \frac{q^2}{[q^-]} \{(2ln\frac{q^-}{\lambda} - \frac{11}{6})(\frac{1}{\epsilon_t} - ln\frac{\mu^2}{q^2}) - (ln\frac{q^-}{\lambda})^2\} \tag{6.14}$$

$$\delta P_2^+ = \frac{g^2}{16\pi^2} T_F \frac{q^- q^+ - q^2}{[q^-]} \frac{4}{6} \frac{1}{\epsilon_t} \tag{6.15}$$

and $\delta P_3^+ = \delta P_4^+ = \delta P_5^+ = 0$.

It is straight-forward to see that the gluon mass is

$$\delta\mu^2 = q^+ \delta P^-_{q^2=q^+q^-=0} + q^- \delta P^+_{q^2=q^+q^-=0}$$

$$\delta\mu^2 = 0 \tag{6.16}$$

Just like in the QED case [3], the massless gauge vector boson remains massless to one-loop, as it should.

We get now the gluon wave-function renormalization by simply using the formula introduced before [3] and putting together the results of the above. We get:

$$Z_A^{-1} = 1 - (\frac{\partial \delta P^-}{\partial q^-})_{q^2=q^+q^-=0} - (\frac{\partial \delta P^+}{\partial q^+})_{q^2=q^+q^-=0}$$

$$Z_A^{-1} = 1 - \frac{g^2}{8\pi^2} \{(\frac{11}{6} C_A - \frac{2}{3} T_F N_F) \frac{1}{\epsilon_t}\} \tag{6.17}$$

and we have obtained the standard covariant result [17], and N_F is as above.

To obtain the vertex renormalization Z_g we observe that we have a simple relation between the gluon wave function renormalization and the first term of the beta function β_0 [18]

$$Z_A^{-1} = 1 - \frac{g^2}{8\pi^2}\beta_0 \frac{1}{\epsilon_t} \tag{6.18}$$

According to [18], the full β-function is

$$\beta(g) = g^2 \frac{\partial Z_{g,1}}{\partial g} = -\beta_0 \frac{g^3}{16\pi^2} - ... \tag{6.19}$$

where $Z_{g,1}$ is the first term in a power series expansion in inverse powers of ϵ_t. We obtain then

$$Z_g = 1 - \frac{g^2}{16\pi^2}(\frac{11}{6}C_A - \frac{2}{3}T_F N_F)\frac{1}{\epsilon_t} \tag{6.20}$$

Notice that we get from this expression a β-function like in the standard covariant calculations [17]. We also seem to have the relation $Z_g = Z_A^{-\frac{1}{2}}$, just like in the background field quantization [18] so that the combination $Z_g Z_A^{\frac{1}{2}}$ remains unrenormalized.

7. Conclusions

We applied the newly invented null-surfaces quantization to heavy quark QCD and calculated, to one-loop in perturbation theory, the renormalization constants $Z_{QCD} = \{Z_Q, Z_q, Z_A, Z_g\}$ and the mass corrections. We find agreement with previous calculations [18] and have the same coefficients for renormalization constants as standard covariant calculations [17]. We also get the same coefficients for the quark mass corrections and we also find that the gluon remains massless to one-loop, as we expect from equal-time calculations[17].

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9. Appendix A: Lorentz Structure of Null-Surfaces Quantization.

What does the Lorentz structure of this quantization look like? Previous work on QED [3] showed us that the generators P^- and P^+ are defined separately along the surfaces Σ_+ and Σ_- respectively. But what is their commutator along, say Σ_+? This is relatively simple to compute: along that surface, $x^+ = 0$, so that P^+ becomes

$$P^+ = \frac{1}{2} \int dx_\perp \mathcal{K}(0, x_\perp) \tag{9.1}$$

Since $\mathcal{K}$ is just $\mathcal{H}$ with the $_+$and $_-$ interchanged, and since along Σ_0 the fields obey the relations [(3.8)] and [(3.11)], we see that

$$\mathcal{K}(0, x_\perp) = \mathcal{H}(0, x_\perp) \tag{9.2}$$

so that $P^+_{\Sigma_+} = P^-_0$ and P^-_0 is P^- with the $x^- = 0$.

Then, the commutator

$$[P^-, P^+]_{\Sigma_+} = [P^-, P^-_0]_{\Sigma_+} = 0 \tag{9.3}$$

It is a straight-forward exercise to see that we also have

$$[P^-, P^+]_{\Sigma_-} = 0 \tag{9.4}$$

as well as

$$[P^-, P^+]_{\Sigma_0} = 0 \tag{9.5}$$

There is a more difficult issue as to what these variables mean: do we have two "times" x^+ and x^- ? Yet the time variable is not an intrinsic property of space-time, so it does not seem that unusual that we have two such coordinates, just like we have several space coordinates. The norm of the space-time four-vector is still the same, and that is a Lorentz invariant.

Regarding the other generators $M^{\mu\nu}$, we do not attempt to construct them in this paper. For a thorough check of Lorentz invariance, we would need to construct them as well and to check that all the Poincaré generators satisfy the algebra. This is beyond the scope of the present work.

It is well-known that Dirac [19] showed that there are three surfaces on which to initialize fields and to quantize fields: the instant-form, used in the usual equal-time quantization; the front-form, used in the usual light-cone quantization calculations, and the point form, used sporadically [20] . A good question is where does this formalism fit in Dirac's three forms. In 1+1-dimensions, it seems that the present work is the same as Dirac's point form, the form which leave invariant the form $x^\mu g_{\mu\nu} x^\nu = \kappa^2$, with $\kappa = 0$. It seems though that in 3+1-dimensions the present surface Σ is different from Dirac's light-cone $x^+x^- - x_\perp^2 = 0$.

10. Appendix B: Interaction Terms in Null-Surfaces Hamiltonians.

We introduce the notation of [9], and use χ for two-component spinors. We present here the forms for $\mathcal{H}_I$. Similar terms exist for $\mathcal{K}_I$, the interaction part of $\mathcal{K}$.

The interaction terms of [(4.11)] are as follows, written in momentum space, like Zhang and Harindranath did in their Table I [9]:

1. Consider three gluons with momenta k_1, k_2 and k_3 respectively, and labels $(i,a),(j,b)$ and (l,c), as in the aforementioned Table. That vertex is identical

to that of [9]:

$$\mathcal{H}_{AAA} = -igf^{abc}\epsilon^i\epsilon^{j*}\epsilon^{l*}\{[(k_2-k_3)^i - \frac{k_1^i}{[k_1^+]}(k_2^+ - k_3^+)]\delta_{jl}$$

$$+[(k_3+k_1)^j - \frac{k_2^j}{[k_2^+]}(k_3^+ + k_1^+)]\delta_{li} + [-(k_1+k_2)^l - \frac{k_3^l}{[k_3^+]}(k_1^+ + k_2^+)]\delta_{ij}\}$$

$$= -igf^{abc}\Gamma_{g0}^{ijl}(k_1,k_2,k_2)\epsilon^i\epsilon^{j*}\epsilon^{l*} \tag{10.1}$$

2. For the four-gluon vertex, we obtain again a similar formula to [9], but we have an overall factor of $\frac{1}{4}$ compared to the similar entry in Table I of [9]. The four gluons are labeled (k_i, a_i) with $i = 1, ..4$:

$$\mathcal{H}_{AAAA} = \frac{g^2}{4}\{f^{a_1a_2b}f^{a_3a_4b}[\delta_{i_1i_3}\delta_{i_2i_4} - \delta_{i_1i_4}\delta_{i_2i_3}] + (2\rightarrow 3) + (2\rightarrow 4)\}$$

$$+\frac{g^2}{2}\{f^{a_1a_2b}f^{a_3a_4b}\delta_{i_1i_2}\delta_{i_3i_4}\frac{(k_1^+ + k_2^+)(k_3^+ + k_4^+)}{[k_1^+ - k_2^+][k_3^+ - k_4^+]} + +(2\rightarrow 3)\} + (2\rightarrow 4)\} \tag{10.2}$$

3. For the heavy-quark-gluon vertex, we label the quark momenta (p_1, α) and (p_2, β), and the gluon momentum k and label (i, a).

$$\mathcal{H}_{Q\overline{Q}A} = +gT^a_{\beta\alpha}[\chi^\dagger\frac{k^i}{[k^+]} - -\frac{1}{2}\frac{\sigma_j p_{2j} - im_Q}{[p_2^+]}\sigma^i - \frac{1}{2}\sigma^i\frac{\sigma_j p_{1j} + im_Q}{[p_1^+]}]\chi\epsilon^{i*}$$

$$+gT^a_{\beta\alpha}[\chi^\dagger\frac{k^i}{[k^+]} - -\frac{1}{2}\frac{\sigma_j p_{2j} + im_Q}{[p_2^+]}\sigma^i - \frac{1}{2}\sigma^i\frac{\sigma_j p_{1j} - im_Q}{[p_1^+]}]\chi\epsilon^{i*}$$

$$+2g\chi^\dagger T^a_{\beta\alpha}\chi\frac{k^i}{[k^+]}$$

$$= T^a_{\beta\alpha}\chi^\dagger\Gamma^i_{Q0}(p_1,p_2,k)\chi\epsilon^{i*} \tag{10.3}$$

In Table I of [9] this vertex was labeled $\Gamma^i_{Q0}(p_1, p_2, k)$. Here, we have our first major difference with Zhang and Harindranath since their vertex is not

Hermitian while ours is. Further, we have an extra piece, the last term, which is not present in [9].

4. The two fermion-two gluon vertex is similar to the [9], but with a small change: we have an extra term, our second, which restores Hermiticity, which - again - was lacking in [9]. Our gluons carry momenta k_i, $i = 1, 2$ and labels (i, b) and (j, c) respectively; the fermions are labeled (p_1, α) and (p_2, β):

$$\mathcal{H}_{Q\overline{Q}AA} = +2ig^2 f_{abc} \chi^\dagger T^a_{\beta\alpha} \frac{(k_1^+ + k_2^+)}{[k_1^+ - k_2^+]} \chi \delta_{ij} \epsilon^i \epsilon^{j*}$$

$$-2ig^2 f_{abc} \frac{1}{[p_1^+]} \chi^\dagger T^a_{\beta\alpha} \frac{k_2^+}{[k_1^+ - k^+)2]} \chi \delta_{ij} \epsilon^i \epsilon^{j*}$$

$$+g^2 \chi^\dagger \frac{(T^a T^b)_{\beta\alpha} \sigma^j \sigma^i}{[p_1^+ - k_1^+]} \chi \epsilon^{i*} \epsilon^j \tag{10.4}$$

5. The four-fermion vertex is also different from that of [9]: we have an extra piece, the second term. Our fermions are labeled (p_1, α), (p_2, β), (p_3, γ) and (p_4, δ):

$$\mathcal{H}_{Q\overline{Q}Q\overline{Q}} = g^2 \chi^\dagger T^a_{\beta\alpha} \chi \chi^\dagger T^a_{\delta\gamma} \chi \{ \frac{4}{[p_1^+ - p_2^+]^2} - \frac{2}{[p_1^+ - p_2^+][p_4^+ - p_3^+]} \} \tag{10.5}$$

6. The mixed heavy-light four-fermion vertex is identical to the heavy quark vertex, with the difference that the light fermions are parametrized by the two-component spinor ρ instead of χ, which we used for the heavy quark:

$$\mathcal{H}_{Q\overline{Q}q\overline{q}} = g^2 \chi^\dagger T^a_{\beta\alpha} \chi \rho^\dagger T^a_{\delta\gamma} \rho \{ \frac{4}{[p_1^+ - p_2^+]^2} - \frac{2}{[p_1^+ - p_2^+][p_4^+ - p_3^+]} \} \tag{10.6}$$

7. As for the light-fermion-gluon vertex, we obtain as above, except that we

don't have a mass term:

$$\mathcal{H}_{q\bar{q}A} = +gT^a\beta\alpha[\rho^\dagger\frac{k^i}{[k^+]} - -\frac{1}{2}\frac{\sigma_j p_{2j}}{[p_2^+]}\sigma^i - \frac{1}{2}\sigma^i\frac{\sigma_j p_{1j}}{[p_1^+]}]\rho\epsilon^{i*}$$

$$+gT^a\beta\alpha[\rho^\dagger\frac{k^i}{[k^+]} - -\frac{1}{2}\frac{\sigma_j p_{2j}}{[p_2^+]}\sigma^i - \frac{1}{2}\sigma^i\frac{\sigma_j p_{1j}}{[p_1^+]}]\rho\epsilon^{i*}$$

$$+2g\rho^\dagger T^a_{\beta\alpha}\rho\frac{k^i}{[k^+]}$$

$$= T^a_{\beta\alpha}\rho^\dagger\Gamma^i_{q0}(p_1, p_2, k)\rho\epsilon^{i*} \tag{10.7}$$

8. Likewise, the light two-quark-two gluon vertex becomes:

$$\mathcal{H}_{q\bar{q}AA} = +2ig^2 f_{abc}\rho^\dagger T^a_{\beta\alpha}\frac{(k_1^+ + k_2^+)}{[k_1^+ - k_2^+]}\rho\delta_{ij}\epsilon^i\epsilon^{j*}$$

$$-2ig^2 f_{abc}\frac{1}{[p_1^+]}\rho^\dagger T^a_{\beta\alpha}\frac{k_2^+}{[k_1^+ - k^+)2]}\rho\delta_{ij}\epsilon^i\epsilon^{j*}$$

$$+g^2\chi^\dagger\frac{(T^aT^b)_{\beta\alpha}\sigma^j\sigma^i}{[p_1^+ - k_1^+]}\rho\epsilon^{i*}\epsilon^j \tag{10.8}$$

9. The light four-quark vertex is:

$$\mathcal{H}_{q\bar{q}q\bar{q}} = g^2\rho^\dagger T^a_{\beta\alpha}\rho\rho^\dagger T^a_{\delta\gamma}\rho\{\frac{4}{[p_1^+ - p_2^+]^2} - \frac{2}{[p_1^+ - p_2^+][p_4^+ - p_3^+]}\} \tag{10.9}$$

10. We also get another light-heavy four-quark vertex,similar to above:

$$\mathcal{H}_{q\bar{q}Q\overline{Q}} = g^2\rho^\dagger T^a_{\beta\alpha}\rho\chi^\dagger T^a_{\delta\gamma}\chi\{\frac{4}{[p_1^+ - p_2^+]^2} - \frac{2}{[p_1^+ - p_2^+][p_4^+ - p_3^+]}\} \tag{10.10}$$

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